UNIVERSITY OF MINNESOTA

ST. ANTHONY FALLS HYDRAULIC LABORATORY

Technical Paper No. 53, Series B

Abrupt Transition from a Circular Pipe to a Rectangular Open Channel

by

Fred W. Blaisdell, Charles A. Donnelly and Kesavarao Yalamanchili
Hydraulic Engineers, USDA, ARS

July 1969

Study conducted by

UNITED STATES DEPARTMENT OF AGRICULTURE
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SOIL AND WATER CONSERVATION RESEARCH DIVISION

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A B S T R A C T

The development of criteria and a generalized procedure for the design of an abrupt transition from a circular pipe to a rectangular open channel are presented.

The rectangular channel must be 1.0 pipe diameters wide. Wider channels cause high waves which reflect from the channel sidewalls, may overtop the sidewalls, and produce severe disturbances in the channel. To permit the pipe to expand, the channel may be widened for a distance not exceeding 0.5 pipe diameters downstream from the pipe exit, and the floor of the channel may be lowered.

The equations developed describe the locations of the water surface elements to within an average of 0.11 pipe diameters of their correct locations. The maximum anticipated location error is ±1.4 pipe diameters. The equations for the envelope curves covering the crests of the sidewall waves, which determine the channel sidewall height, provide an average freeboard of 0.08 pipe diameters and a maximum freeboard of 0.31 pipe diameters. When the envelope equations are used only 2 percent of the wall waves will overtop the sidewalls, the maximum overtopping being 0.04 pipe diameters. The average depth of flow—the depth at the wave nodes—is predicted by the equations to within a maximum deviation of +0.13 and -0.06 pipe diameters of the observed depths. The average depth at the nodes is predicted by the equations within 0.01 pipe diameters of the observed average depth.

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NOMENCLATURE

A	coefficient in Eq. 6, dimensionless
d	depth of flow, in feet
d	differential
d_{aN}	average depth of flow as computed by Eqs. 11 and 15, in feet
d_N	depth of water surface elements in the transition channel, in feet
d_{wN}	wave height above the computed average depth of flow d_{aN}, in feet
D	pipe diameter, in feet (unless otherwise defined)
f	Darcy-Weisbach friction factor, dimensionless
f_e	psuedo Darcy-Weisbach energy loss factor, dimensionless
g	acceleration due to gravity = 32.2 feet per second per second
h_e	energy head, in feet of water, $= d_N + \left(\frac{Q}{d_N D}\right)^2 \frac{1}{2g}$
h_ℓ	energy loss, in feet of water
L_p	length of parallel-walled section, in feet, in the transition developed by C. D. Smith
N	number of the water surface element. Used as a number or a subscript. $N = 1, 5, 9, 13, \ldots$ designate sidewall waves $N = 3, 7, 11, 15, \ldots$ designate centerline waves Even values of N designate nodes where the transverse water surface is level
Q	discharge, in cubic feet per second
R	hydraulic radius, in feet
$\mathbb{R}$	Reynolds number, dimensionless, $= \frac{4VR}{\nu}$
S	pipe slope, sine
t	time, in seconds
t_p	pipe wall thickness, in feet
V_p	velocity in the pipe, in feet per second
x	horizontal distance from the pipe exit, in feet
x_N	horizontal distance from the pipe exit to surface element N, in feet
x_Δ	distance between x_2 and x_N, in feet

y	vertical distance from pipe invert at its exit (unless otherwise defined), in feet
α	slope angle of pipe, in degrees = tan ($\sin^{-1}$ S)
Δ	increment of . . .
λ	wave length, in feet
ν	kinematic viscosity, in feet^2 per second
π	3.14159
$\langle\ \rangle$	indicates that the quantity within the pointed brackets is zero for negative values

ABRUPT TRANSITION FROM A CIRCULAR PIPE TO A RECTANGULAR OPEN CHANNEL[±]

INTRODUCTION

An outlet for closed conduit spillways consisting of a horizontal cantilevered channel of rectangular cross section having a flip bucket or flip sill at the downstream end was proposed by the Engineering Division, Soil Conservation Service (SCS), U.S. Department of Agriculture. It was reasoned that the flip sill would throw the water upward and away from the exit end of the cantilevered channel and spread it out. The excavated or self-formed plunge pool would thus be further from the cantilevered channel exit and the toe of the dam. Also, the falling jet would impinge on the tailwater surface at a steeper angle to facilitate the vertical dissipation of energy, and the lesser horizontal component of the velocity would reduce the strength of the horizontal eddies that widen the plunge pool and eat into the dam toe.

Closed conduit spillways frequently are made of circular pipe, whereas the proposed cantilever is rectangular in cross section. Therefore, a transition between the circular barrel and the rectangular cantilever is required. Because the transition and the flip sill can be studied separately, the test program was divided into two parts: (1) the development of a transition between a circular spillway conduit and a rectangular open channel, and (2) a study of the flip sill.

The test results and generalized rules for the design of an abrupt transition between a circular pipe and a rectangular open channel are reported here.

REQUIREMENTS

The SCS imposed several practical limits on the transition design:

1. It was suggested that the transition be abrupt.

2. To facilitate installation, the outside of the pipe should be no closer than 3 in. to the inside walls and floor of the transition. In other words, the inside width of the transition should not be less than the pipe diameter D, plus two times the pipe wall thickness t_p, plus two times 3 in. or $(D + 2t_p + 6'')$, and the floor of the transition must be a minimum of $(t_p + 3'')$ below the pipe invert. The specific dimensions were generalized in terms of the pipe diameter D, with consideration given to the pipe sizes and wall thicknesses used by SCS. The minimum transition width was set at 1.5 D and the minimum distance of the transition floor below the pipe invert was set at 0.25 D. These generalized dimensions simulate a 24-in. diameter pipe having a 3-in. wall and are greater than the minimums required for larger pipes.

[±]Agricultural Research Service Report No. 41-304-141.

3. To allow for settlement of the dam fill, the pipe must be free to move longitudinally a maximum of 6 in. This is equivalent to 0.25 D for a 24-in. diameter pipe.

4. The flow in the transition must have a relatively level transverse surface at the location of the flip sill.

PREVIOUS WORK

The only available report presenting the results of research on an abrupt transition from a circular pipe to a rectangular open channel was published by the Canada Department of Agriculture in 1954*. One object of this research was to develop a flaring transition between a circular pipe and a stilling basin. C. D. Smith reports:

> After the first series of tests had been run, it was evident that it was undesirable to have the straight wall flare begin immediately at the end of the circular pipe. Invariably flow separation occurred at the walls at the start of the transition. This resulted in the formation of a high local shock wave on the wall further along in the transition. It was found that this could be eliminated by the use of a short parallel section between the pipe and the start of the transition. In this parallel section the cavities which normally exist under the jet (when in its circular form) are allowed to fill, and at the start of the transition the jet is essentially rectangular and two dimensional.

The required length of the parallel-walled section L_p is given as:

$$\frac{L_p}{D} = 0.1 \frac{Q}{D^{5/2}}$$

where Q is the discharge in cubic feet per second. The width of the parallel-walled section is equal to the pipe diameter D. The transition begins its flare at the end of the parallel-walled section.

Two conclusions can be drawn from the research conducted by Smith: (1) a parallel-walled section equal in width to the pipe diameter is required to prevent high local shock waves, and (2) although the length of the parallel-walled section gave good performance when used in conjunction with flaring transition sidewalls, this length may not be correct for other transition geometries.

At the request of Edwin Freyburger, then in charge of the Design and Construction Section, Engineering and Watershed Planning Unit, Soil Conservation Service, Milwaukee, Wisconsin, a short series of tests was run in November, 1957, on an abrupt transition from a circular pipe to a rectangular chute. F. W. Blaisdell reported in a letter to Mr. Freyburger dated November 19, 1957, that:

> 1. The chute width should be equal to the pipe diameter.
>
> 2. A parallel sidewall section having a floor slope identical to the pipe slope should be used. The sidewalls should be spaced one pipe diameter apart. The length of this section apparently should be, after the Canadian experiments, $0.1\,Q/D^{5/2}$.
>
> 3. The use of fillets is beneficial but they do not seem necessary if the parallel sidewall section is used.

---

*Smith, C. D., "Hydraulic Design of Outlet Transition and Stilling Basin for Single Barreled Conduits," Design Bulletin No. 2, Canada Department of Agriculture, Regina, Saskatchewan, Canada, May, 15, 1954, 15 pp.

The floor of the chute for these tests was tangent to the invert of the pipe. The floor had a slope equal to the pipe slope in the parallel-walled section, and a parabolic curve to fit the jet trajectory between the parallel-walled section and the steeply sloping chute.

The results of the tests conducted by the Prairie Farm Rehabilitation Administration and the Agricultural Research Service were incorporated into Soil Conservation Service Standard Drawing No. 3-E-45341, part of which is reproduced as Fig. 1. Fig. 1 shows that:

1. The chute width is equal to the pipe diameter.

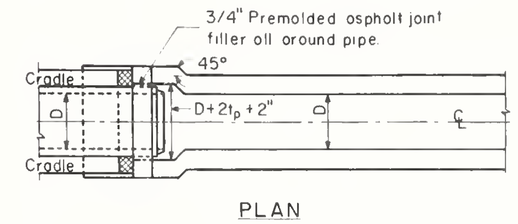
2. An expansion section is provided between the pipe and the chute.

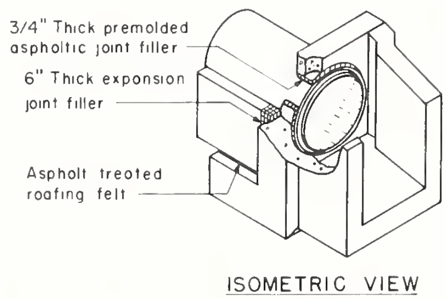
3. The pipe is set above the chute floor to provide for pipe expansion.

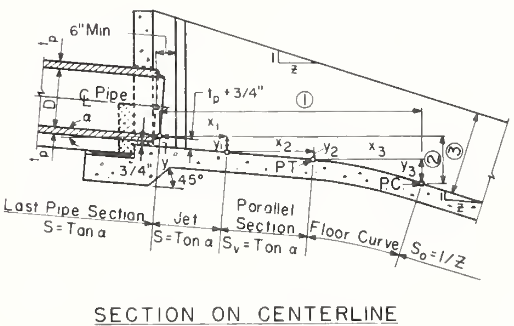
4. A length of chute having the same slope as the slope of the pipe is provided between the pipe exit and the point where the jet strikes the chute floor.

5. A parallel-walled section is provided which has a floor slope equal to the pipe slope and a length recommended as a result of the reported tests.

6. A floor curve is provided between the parallel section and the steeply sloping chute.







(1) Horizontal distance from invert at outlet end of pipe to PC
(2) Difference in elevation between these 2 points
(3) Normal height of chute sidewalls at PC

NOTE: THIS PROCEDURE APPLIES FOR VALUES OF $\frac{Q}{D^{5/2}} \leq 20$

GENERAL FORMULAS	REF.
JET $y = (t_p + \frac{3''}{4}) \sec \alpha$, slope length $= x_1 \sec \alpha$, $y_1 = y + x_1 \tan \alpha$ $x_1 = \sqrt{\frac{2 v_p^2 \cos^2 \alpha y}{g}}$	1
PARALLEL SECTION slope length $= 0\ 1 \frac{Q}{D^{3/2}}$ $\quad y_2 =$ (slope length) $\sin \alpha$ $x_2 =$ (slope length) $\cos \alpha$	2
FLOOR CURVE $x_3 = 50 (\frac{1}{z} - \tan \alpha)$ $\quad y_3 = 25 (\frac{1}{z^2} - \tan^2 \alpha)$ Note: Coordinates of any point between P.T. and P.C. are given by equation: $y = 0\ 01 x^2 + S_v x$	3
WIDTH OF CHUTE SIDEWALLS (inside) = D	2
MINIMUM HEIGHT OF CHUTE SIDEWALLS height = D + 0 5'	
REFERENCES (1) National Handbook Section 5 - Hydraulics page 5 6-2 (2) See footnote *, page 1 (3) National Handbook Section 14 - Chute Spillways page 2 120, 2.141, 2 142, 2 143 and 2.144	

Fig. 1 - Soil Conservation Service Transition Layout

These results of previous research, although limited, provide an indication of the problems to be anticipated in developing general criteria for the design of an abrupt transition from circular to a rectangular open channel.

TEST APPARATUS

Figure 2 shows the test apparatus. The water for the experiments is obtained from the laboratory main supply channel. The offtake from the main supply channel is a 6-in. steel pipe. A 4-in. steel pipe, 10 in. long, was welded into the 6-in. supply line, forming a reducing tee.

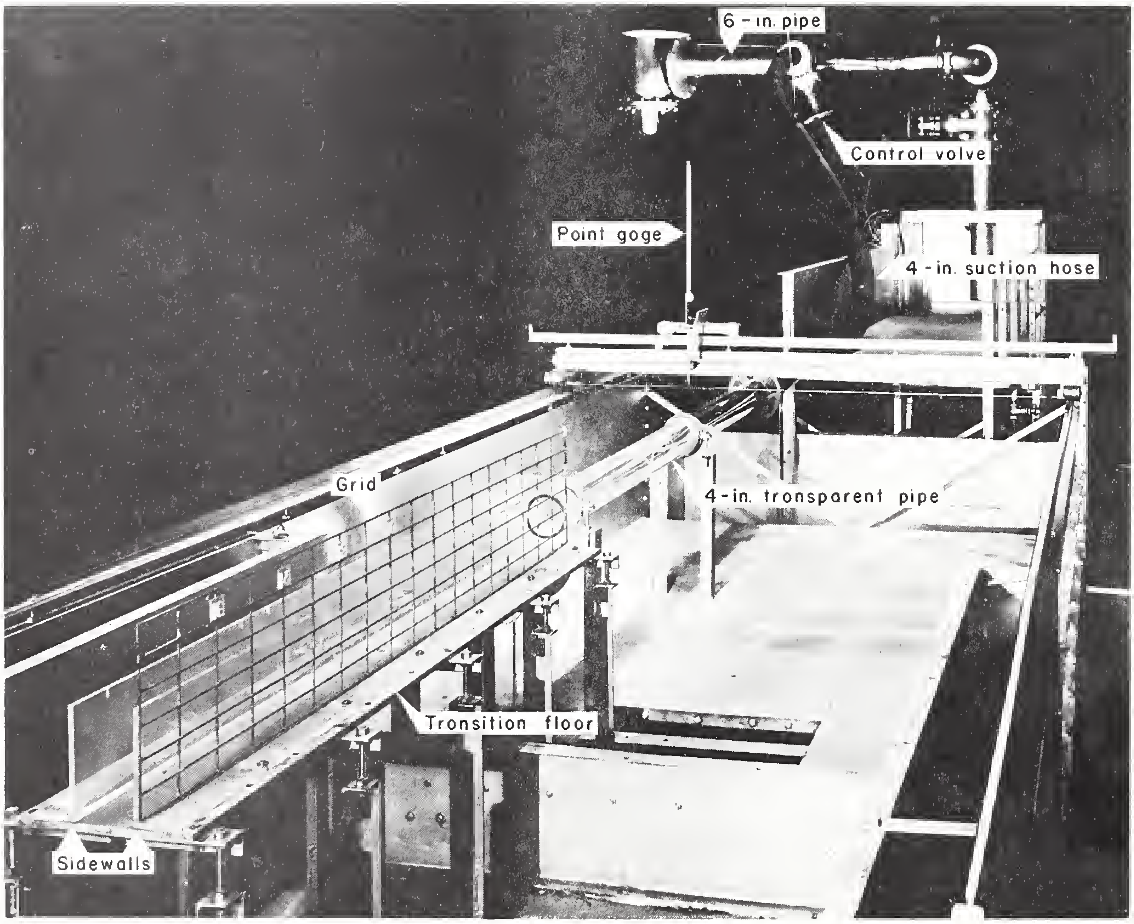


Fig. 2 - Test Apparatus

The rate of flow is indicated by a water manometer connected to piezometer taps located in zones of high and low pressures on opposite sides of the 4-in. pipe close to its junction with the 6-in. pipe. The calibrated relationship between the pressure difference and the discharge is used to determine the rate of flow through the transition. The 4-in. steel pipe terminates in a 4-in. flow control valve.

A 4-in. rubber suction hose, 16.5 ft. long, connects the 4-in. valve with the 4-in. diameter by 10-ft. (30 D) long pipe located upstream of the transition. This pipe is made of transparent plastic so that the flow approaching the transition can be observed.

The transition floor is an aluminum plate, ground to a plane surface. The aluminum plate is 1/2 in. thick, 12 in. (3 D) wide, and 72 in. (18 D) long. The transition floor is supported on threaded rods so it can be leveled and its height adjusted.

The transition sidewalls are clear plastic, 7 in. (1.75 D) high. They can be moved transversely and longitudinally to obtain various combinations of transition width and expansion section length.

A grid, made of 1/8-in. brass rods spaced 4 in. (1 D) longitudinally and 2 in. (0.5 D) vertically, is placed along one sidewall so that a photographic record can be made of the wave lengths and heights. Zero for the longitudinal dimension is the pipe exit; zero for the vertical dimension is the transition floor. Distances are numbered in pipe diameters.

A point gage that traverses longitudinally and transversely was used to determine levels anywhere in the transition. A rake consisting of five teeth 1/4 in. wide and spaced 5/8 in. apart was substituted for the point for the tests of the 1.0 D-wide transition to make it easier to determine the locations and depths of the water surface configuration components and to improve the precision of the measurements.

DESCRIPTION OF FLOW

A knowledge of how the water discharges from the pipe and impinges and spreads on the transition floor is necessary in order to understand the performance of the transition.

The water leaves the pipe as a circular jet, as can be seen in Fig. 3. Beyond the pipe exit, the jet is acted on by gravity and, if falling freely, assumes a parabolic trajectory. The trajectory of the bottom of the jet has the equation

$$y = x \tan \alpha + \frac{g}{2} \frac{x^2}{V_p^2 \cos^2 \alpha} \tag{1}$$

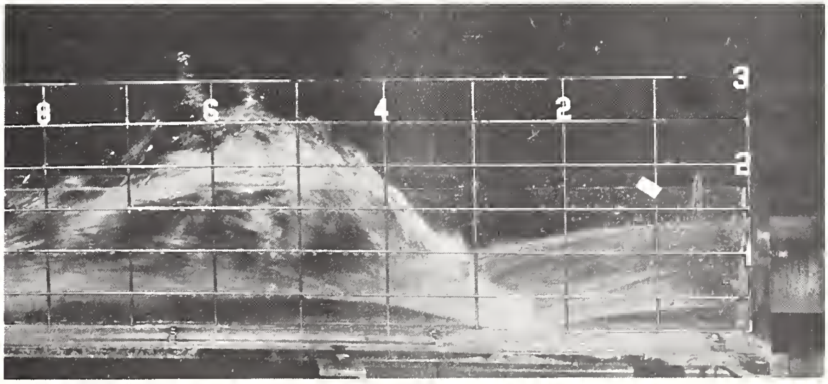


Fig. 3 - Jet Leaving Pipe has a Parabolic Trajectory
Width = 1.5 D; y/D = 0.25; $Q/D^{5/2}$ = 15.13.

where y is the vertical distance below the pipe invert, x is the horizontal distance from the pipe exit, V_p is the velocity at the pipe exit, and α is the slope angle of the pipe, in degrees.

In terms of the pipe diameter D and the generalized discharge $Q/D^{5/2}$

$$\frac{y}{D} = \frac{x}{D} \tan \alpha + \frac{g \pi^2}{32 \left(Q/D^{5/2}\right)^2 \cos^2 \alpha} \left(\frac{x}{D}\right)^2 \tag{2}$$

If the pipe is horizontal, as it was during the experiments reported here, $\alpha = 0$ and

$$y = \frac{g}{2} \frac{x^2}{V_p^2} \tag{3}$$

$$\frac{y}{D} = \frac{g \pi^2}{32 \left(Q/D^{5/2}\right)^2} \left(\frac{x}{D}\right)^2 \tag{4}$$

and

$$\frac{x}{D} = \frac{4}{\pi} \sqrt{\frac{2}{g}} \frac{Q}{D^{5/2}} \sqrt{\frac{y}{D}} \tag{5}$$

There is no pressure within the freely falling jet so, unless there are other influences, the jet will maintain its circular form until it strikes the transition floor. In other words, the jet will not begin to change shape or begin to spread until it impinges on the floor of the transition. This is shown in Fig.4, where $Q/D^{5/2} = 11.76$ and the value of y/D for the bottom of the jet is 0.25. The Eq. 5 value of x/D, 1.87, agrees well with the point of impingement of the bottom of the jet on the transition floor. For the top of the jet, $(y + D)/D = 1.25$ and the Eq. 5 value of x/D is 4.17. This distance is close to the projection of the top surface of the jet to the transition floor. On the basis of these comparisons, it is apparent that Eqs. 1 to 5 predict the trajectory of the freely falling jet.

The initial contact of the jet with the transition floor (at about $x/D = 1.87$ in Fig. 4) causes the jet to begin to spread laterally. This lateral spreading continues until the flow strikes the transition channel sidewalls. The transverse component of the velocity then causes the flow to climb the sidewalls. This action is shown in Fig. 4.

The water that has climbed the sidewalls ($N = 1$ in Fig. 5)* then falls toward the transition floor. This creates a cross wave which, when combined with the downstream flow, forms the diagonal wave shown in Fig. 5. The diagonal wave height is augmented where the waves originating at opposite walls cross at the transition centerline ($N = 3$ in Fig. 5). The diagonal waves then continue across the channel and are reflected from the walls opposite those at which they originated ($N = 5$ in Fig. 5). The reflections ($N = 1$, 5 and 9 in Fig. 5) and crossings ($N = 3$, 7 and 11 in Fig. 5) are repeated over and over so that the resulting water surface displays a diamond-shaped pattern. This pattern can be seen in Fig. 5. The waves decrease in height with distance downstream, but their effect continues for a considerable distance.

---

* N is the number of the water surface configuration component—the number of quarter wave lengths from the pipe exit. The first sidewall wave crest is assigned $N = 1$ and for succeeding sidewall wave crests $N = 5, 9, 13, \ldots$. The centerline crossing waves are assigned $N = 3, 7, 11, \ldots$. The points at which the transverse water surface is essentially horizontal, which are here called "nodes," are even numbers—$N = 2, 4, 6, 8, 10, \ldots$

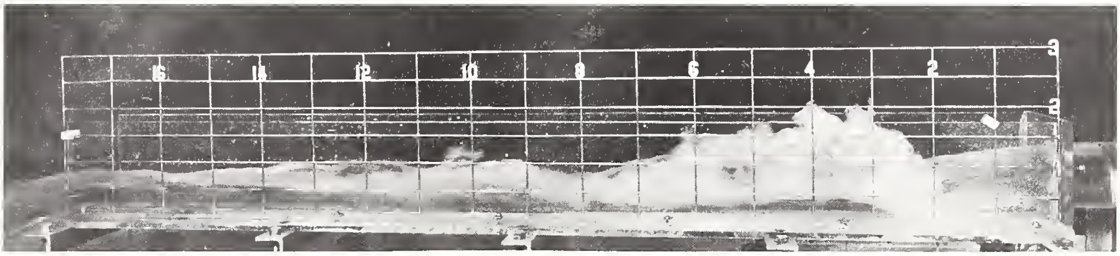


Fig. 4 - Jet Impinges on Floor
Width = 1.5 D; $y/D = 0.25$; $Q/D^{5/2} = 11.76$.

The water surface is almost horizontal at the nodes, which are located approximately midway between the peaks of the sidewall and the center crossing waves, (N = 2, 4, 6, 8 and 10 in Fig. 5). Therefore, since a node meets the criterion of item 4 of the section on REQUIREMENTS that the water surface be relatively level transversely at the location of the flip sill, a node may be the point at which to locate a flip sill or other appurtenance.

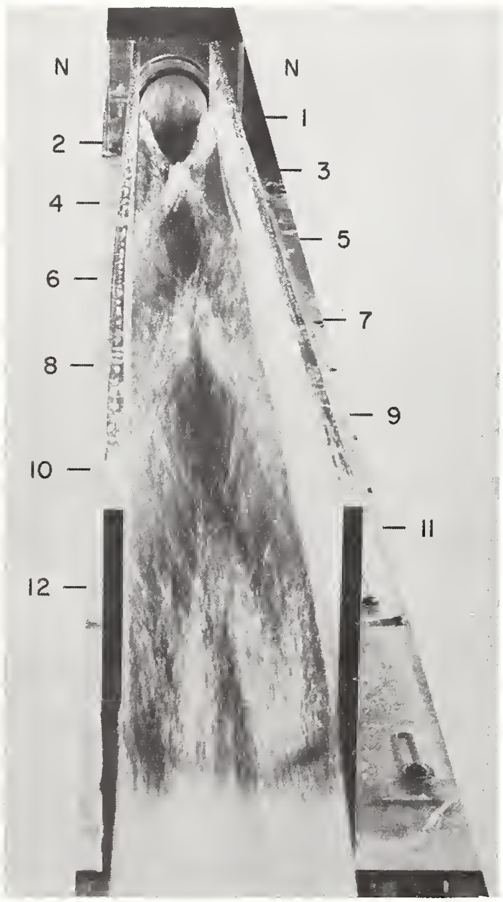


Fig. 5 - Diagonal Wave Pattern in Downstream Channel
Width = 1.0 D; $y/D = 0.25$; $Q/D^{5/2} = 10.5$.

This description of the flow in abrupt transitions from circular to rectangular cross sections illustrates the problems and limitations that must be overcome in this type of transition to insure its satisfactory performance.

TEST PROGRAM

Four independent variables required tests to determine their effect on the transition performance and to develop design criteria. These variables are: (1) the width of the transition channel relative to the pipe diameter, (2) the transition channel floor elevation relative to the pipe invert, (3) the expansion of the barrel, and (4) the discharge. Two dependent variables are: (1) the required transition channel sidewall height to contain the flow, and (2) the length of transition channel required to insure satisfactory flow conditions for satisfactory performance of the flip sill or other appurtenance. The length of the transition channel will be determined during the tests of the flip sill; here, the locations and heights of the water surface components will be determined.

As noted in the section on REQUIREMENTS, the minimum width of the transition was established as 1.5 pipe diameters (1.5 D) for practical reasons. This width was tested. However, the experiences cited previously suggested that a transition this wide would produce poor flow conditions. For this reason, additional tests were scheduled using transition widths of 1.25 D and 1.0 D.

The floor of the transition must be set lower than the pipe invert to permit the pipe to clear the

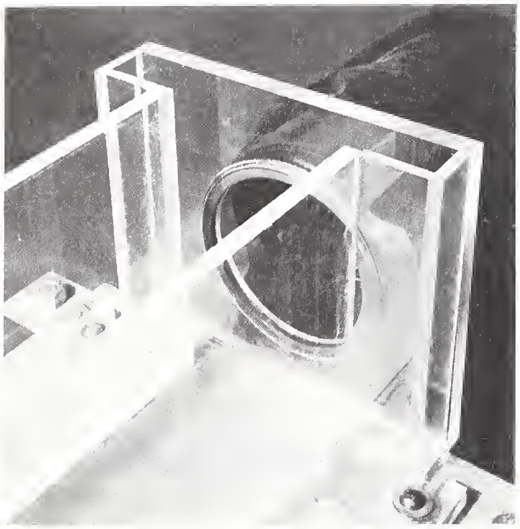


Fig. 6 - Widened Channel Section to Permit Pipe to Expand

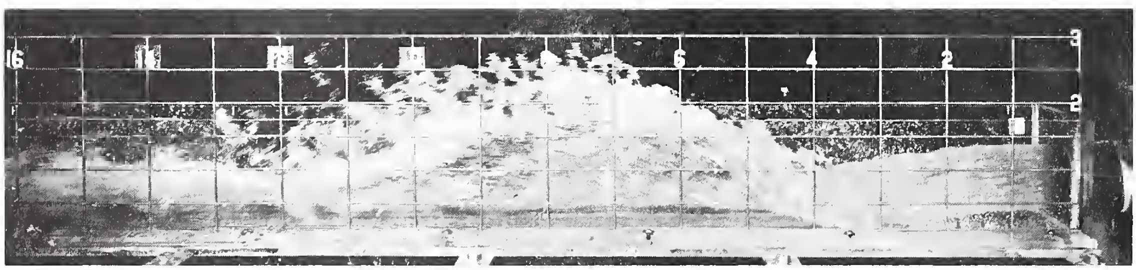
Channel width = 1.0 D; y/D = 0.25.

floor when it moves into the expansion section of the transition. A study of the wall thicknesses of commercial concrete pipe and the required clearance between the pipe and the transition floor showed that the greatest depression of the floor required is 0.25 D below the pipe invert and the minimum depression is 0.10 D. The tests were performed with the floor level with the pipe invert and 0.05 D, 0.10 D, 0.15 D, 0.20 D, 0.25 D, and 0.30 D below the pipe invert.

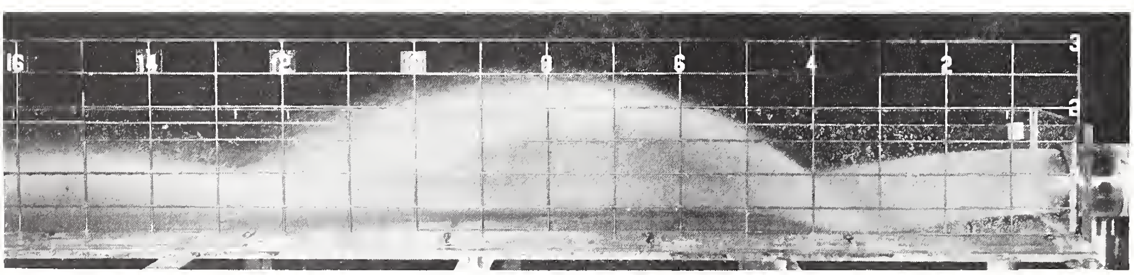
The pipe and the transition floor were horizontal for all tests reported here.

A section is required at the transition entrance to permit the pipe to expand when the transition width is less than 1.5 D. Practical considerations dictated a minimum expansion length of 0.25 D. Additional tests were made to determine the maximum length of the expansion section that could be used without adversely affecting the transition performance. The expansion sections tested were 0.25 D, 0.375 D, 0.5 D, and 0.625 D long. Their width was 1.8 D. A typical expansion section is shown in Fig. 6.

The transition must function satisfactorily at all discharges. To make the discharge Q a generalized dimension, it was divided by $D^{5/2}$ because for a given value of this ratio all pipe diameters give similar flow patterns. The range of $Q/D^{5/2}$ tested was from 1.5 to 20.



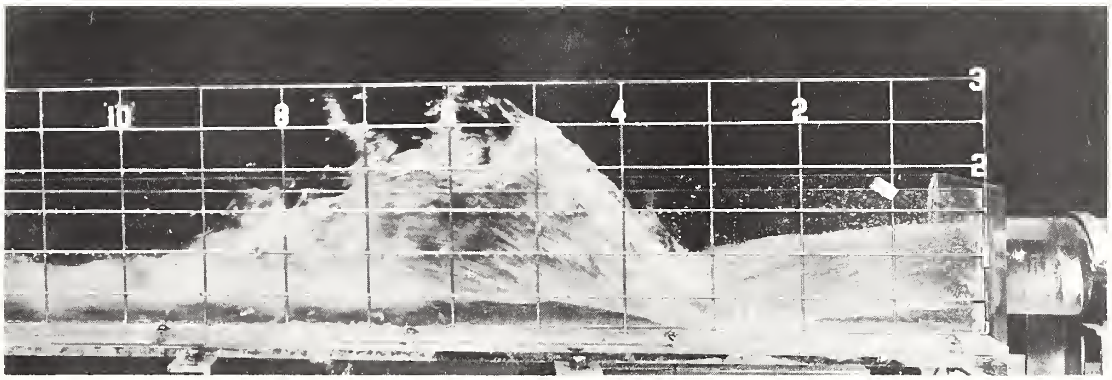
(a) Exposure is 1/250 second.



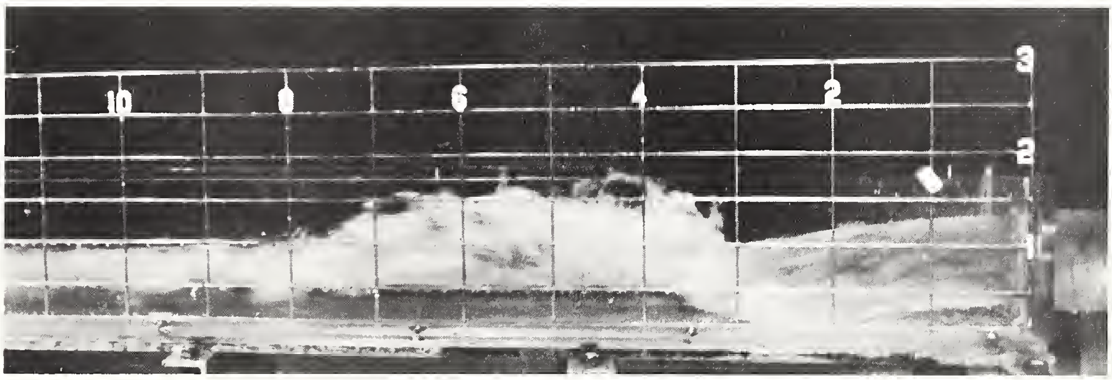
(b) Exposure is 1/2 second.

Fig. 7 - Splash in the Transition Channel
Width = 1.5 D; y/D = 0.25; $Q/D^{5/2}$ = 14.0.

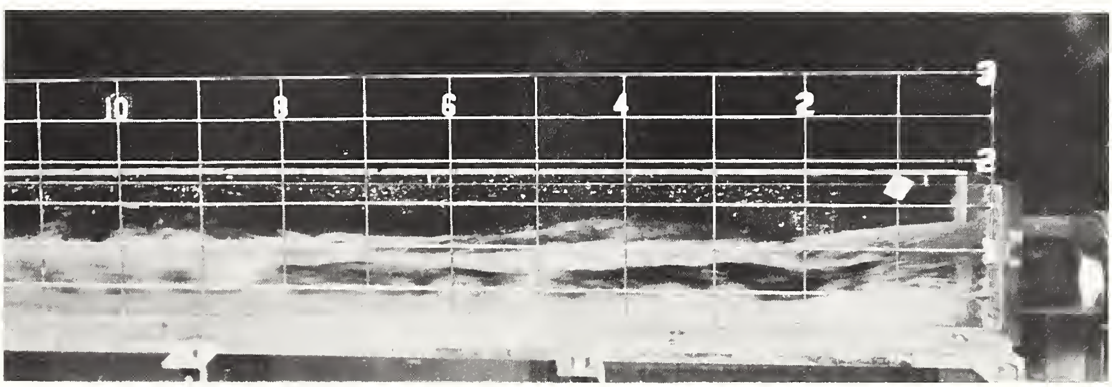
Measurements of the water surface elevations in the transition were taken. However, photographic recording of the flow in the transition proved adequate to evaluate the transition performance when the water surface was rough. Short-exposure photographs were taken to show the nature of the splash (see Fig. 7a). Long-exposure photographs were taken to indicate the average maximum height of the splash and the sidewall height required to contain the splash (see Fig. 7b).



(a) Width is 1.5 D.



(b) Width is 1.25 D.



(c) Width is 1.0 D.

Fig. 8 - Effect of Transition Channel Width on the Performance
$y/D = 0.25$; $Q/D^{5/2} = 14.87$.

After the tests had been completed and the results had been analyzed, advantage was taken of an opportunity to study the effect of scale on the findings. These tests were made using a 36-in. pipe in a 9-ft. (3 D) wide concrete channel. This pipe is 9 times the size of the 4-in. pipe previously used. Since the widest channel previously tested was 1.5 D wide, tests were made on the 4-in. pipe duplicating the large-scale tests. The transition floor for the comparison tests was 0.21 D below the pipe invert. The flow, $Q/D^{5/2}$, was 9.04 and 12.6 for both pipe sizes.

RESULTS OF TESTS

The experiments determined, for a wide range of discharges, the effect on the performance of the abrupt circular-to-rectangular transition of (1) the distance between the sidewalls relative to the pipe diameter, (2) the elevation of the transition floor with respect to the pipe invert, and (3) the length of the pipe expansion section. Design criteria are presented for these three dimensions and the sidewall height. Equations are developed to define the longitudinal positions of the wave elements, the depth of flow at the nodes and the average depth of flow, and the heights of the sidewall waves. The data used to develop the design criteria and the equations are listed in Table 2, appendix.

Distance Between the Sidewalls

The section on DESCRIPTION OF FLOW describes the spreading of the jet to the sidewalls and the waves that form when the flow is reflected from the sidewalls.

The width of the transition channel greatly affects the wave heights and the splash. This is shown in Fig. 8 where the height at the sidewall of the initial wave is 3.0 D for the 1.5 D-wide transition, 2.0 D for the 1.25 D-wide transition, and 1.3 D for the 1.0 D-wide transition. No numerical data are necessary to show the effect of the transition channel width on the wave height, nor were any used. The superior performance of the 1.0 D-wide transition is obvious from casual observation and from photographs similar to those shown in Fig. 8.

The wave height is lower in the narrower transitions because the falling jet suppresses the rising wave at the wall. In the 1.5 D-wide and 1.25 D-wide transitions, there is sufficient space between the 1.0 D-wide falling jet and the wall for the wave to move upward past the jet. In the 1.0 D-wide transition, the jet occupies the full width of the transition so that the rising wall wave cannot bypass the falling jet. The result is a much smoother water surface in the transition. This superior performance of the 1.0 D-wide transition occurs at all discharges.

It is recommended that the transition channel width be 1.0 D.

Expansion Section

A special expansion section is required to permit the pipe to expand when the transition channel width is equal to the pipe diameter. This expansion section is located at the pipe exit and precedes the the transition proper.

The transition channel floor was 0.25 D below the pipe invert during the tests to determine the effect of the expansion section on the performance.

If the expansion section is short enough, the jet leaving the pipe will shoot through the expansion section and into the transition channel without causing flow disturbances. This is because the jet leaving the pipe is free-falling and therefore does not expand immediately. The appearance of the jet as it passes through the expansion section is illustrated in Fig. 9. There it can be seen that the jet does not im-

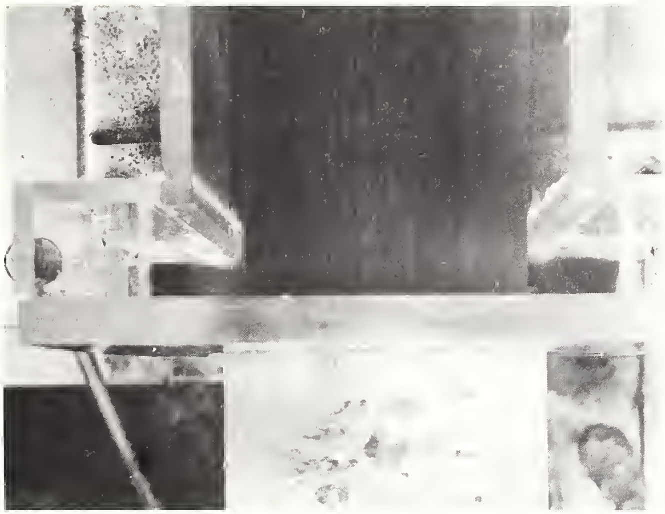


Fig. 9 - Appearance of the Jet as it Leaves the Pipe, Passes Through the Expansion section, and Enters the 1.0 D Wide Transition Channel

At a high discharge ($Q/D^{5/2}$ = 10.5) the jet passes through the expansion section and into the transition channel without spreading. Expansion section is 0.25 D long by 1.8 D wide.

pinge on the downstream walls of the expansion section and that the expansion section does not cause additional disturbance to the flow in the transition channel. The tests showed that satisfactory flow conditions were obtained for expansion section lengths of 0.25 D, 0.375 D, and 0.50 D. However, when the expansion section was lengthened to 0.625 D, the jet spread out, struck the downstream face of the expansion section, and caused high waves and excessive disturbance in the transition channel.

The performance of the expansion section is best at high discharges but is satisfactory at low discharges. At high discharges, the jet from the pipe exit shoots through the expansion section and into the transition channel without causing disturbances in the expansion section or in the transition channel. This is illustrated in Fig. 9. At low discharges, the jet spreads transversely and the edges of the jet strike the downstream walls of the expansion section as shown in Fig. 10. However, because the velocity is low at these low discharges, the disturbances in the expansion section and the waves that form within the transition channel are small, and they are tolerable.

The width of the expansion section tested was 1.8 D. Based on SCS criteria and a 24-in.-diameter pipe, the minimum width would be about 1.5 D. However, the width tested is not important because the width of the expansion section has no effect on the flow conditions in the transition channel. The range of discharge tested was from $Q/D^{5/2}$ = 1.5 to 20. At high discharges there was little or no water in the expansion section. At low discharges only a small part of the flow entered the expansion section and this did not cause any problems. Therefore, the expansion width is not controlled by hydraulic considerations. Practical considerations indicate that the expansion section width must be greater than the outside diameter of the pipe ($>(D + 2t_p)$) in order to permit the pipe to expand.

Fig. 10 - Appearance of the Jet as it Leaves the Pipe, Passes Through the Expansion Section, and Enters the 1.0 D Wide Transition Channel

At a low discharge ($Q/D^{5/2}$ = 5.4) the jet edges spread into the expansion section but do not cause intolerable flow conditions. Expansion section is 0.25 D long by 1.8 D wide.

The tests show that: (1) the minimum length of the expansion section is determined by the anticipated pipe expansion; (2) the maximum length of the expansion section should not exceed 0.5 D in order to avoid excessive disturbances in the expansion section and in the transition channel; and (3) the expansion section width is not controlled by hydraulic considerations, but should be greater than the outside diameter of the pipe so there will be no constraint on the longitudinal movement of the pipe.

Water Surface Configuration

The water surface configuration has been described qualitatively in the section entitled DESCRIPTION OF FLOW and is illustrated in Fig. 5. Elements comprising the water surface configuration—the sidewall wave, the centerline crossing waves and the nodes (the cross sections where the water surface is horizontal)—will now be described quantitatively. This quantitative description is limited to the recommended 1.0 D-wide transition having an expansion section 0.25 D long and a horizontal transition floor located from 0.00 D to 0.30 D below the pipe invert.

The precision of this quantitative description is variable. For low discharges, the wave peaks and water surface are relatively well defined and there is a minimum of splash. For high discharges, splash makes it difficult to determine the wave height and the long wave crests make it difficult to determine the exact position of the wave crest. These comments should be kept in mind when evaluating the agreement of the experimental data with the equations developed from the data and the agreement of the plotted data with the curves which represent the equations.

Distance from Pipe Exit to Surface Elements

Distances from the pipe exit to the crests of the wall waves (N odd: 1, 5, 9, . . .), to the crests of the diagonal waves where they cross at the centerline (N odd: 3, 7, 11, . . .), and to the nodes where the water surface is approximately horizontal (N even: 2, 4, 6, . . .) are plotted against $Q/D^{5/2}$ for each y/D tested in Figs. 11, 12, 13, 14, 15, 16 and 17. The plotted data pertain to full pipe. Full pipe flow was obtained in the model when $Q/D^{5/2}$ exceeded 4.5 to 5.2 on increasing discharges and was maintained down to $Q/D^{5/2} = 3.8$ for decreasing discharges.

Families of straight lines have been adjusted to the data plotted in Figs. 11 to 17 to give the best fit for all the data. Exact agreement of the curves with the experimental data cannot be expected because judgment is required in making the measurements, especially at the higher discharges where the water surface is poorly defined and there is considerable splash. However, the experimental data generally agree well with the curves shown. Some check tests were made for those tests where the original data deviated appreciably from the curves and were suspect. The check test data fall close to the curves as drawn.

The curves as drawn have the equation,

$$\frac{x_N}{D} = A\left(\frac{Q}{D^{5/2}} + 10\sqrt{\frac{y}{D}}\right) - 4.5\sqrt[4]{\frac{y}{D}} \tag{6}$$

where x_N is the horizontal distance from the pipe exit to the surface configuration element, and y is the vertical distance from the floor of the transition to the pipe invert and is positive. Eq. 6 is valid only when the velocity in the transition is supercritical.

(Continued on page 20)

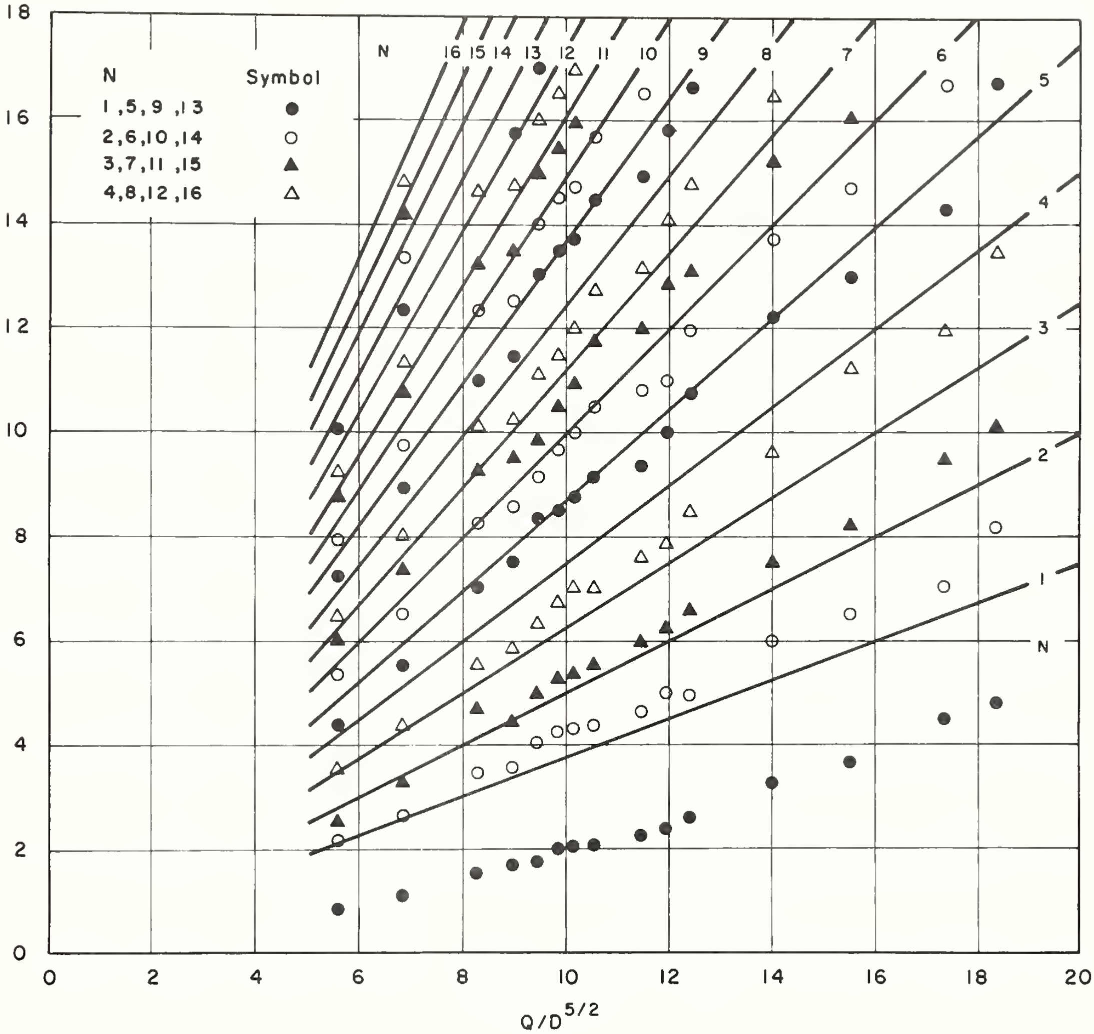


Fig. 11 - Distance from Pipe Exit to Water Surface Components
Pipe is full. Transition is 1.0 D wide. Expansion Section is 0.25 D long. y/D = 0.00.

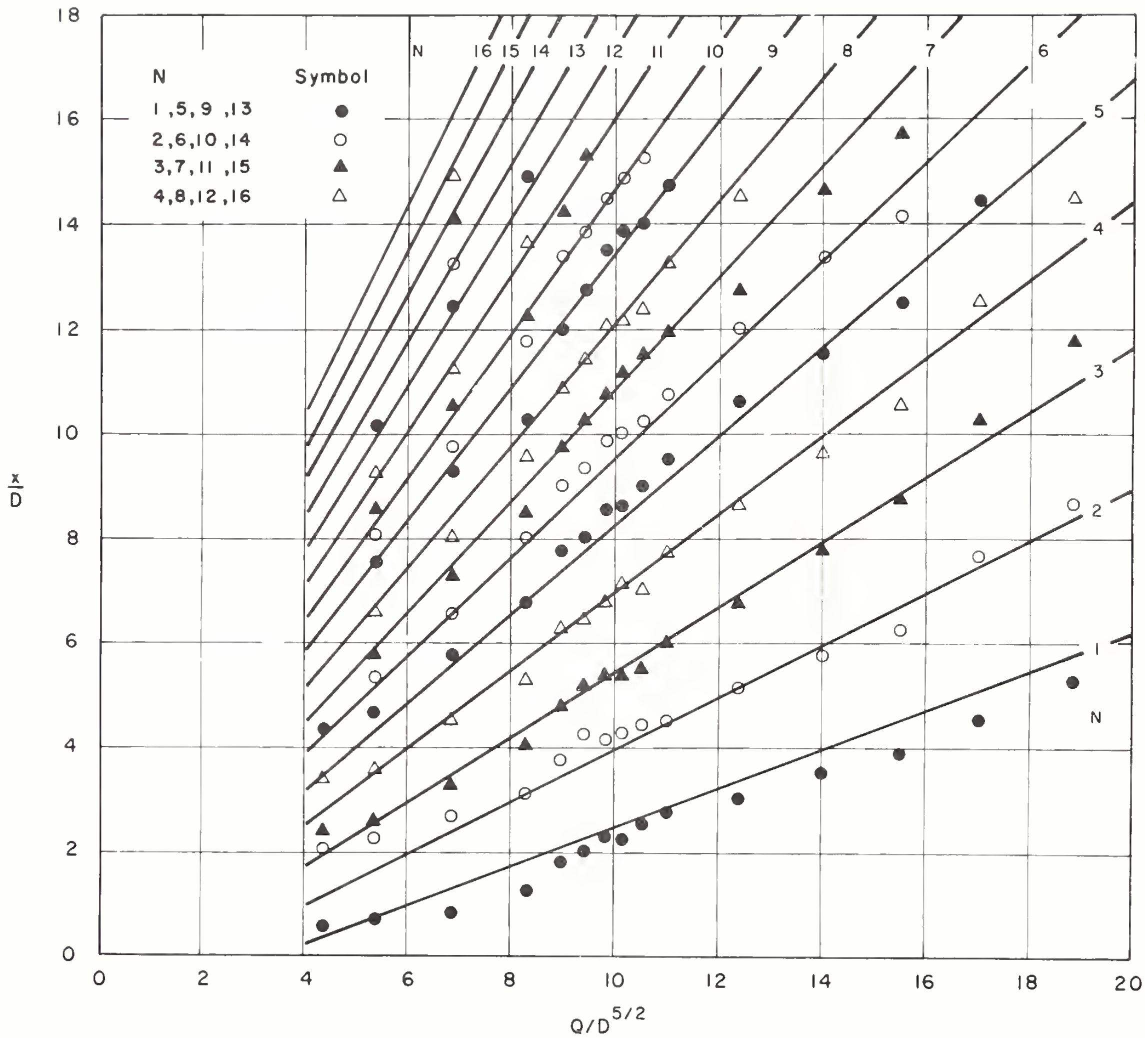


Fig. 12 - Distance from Pipe Exit to Water Surface Components
Pipe is full. Transition is 1.0 D wide. Expansion section is 0.25 D long. y/D = 0.05.

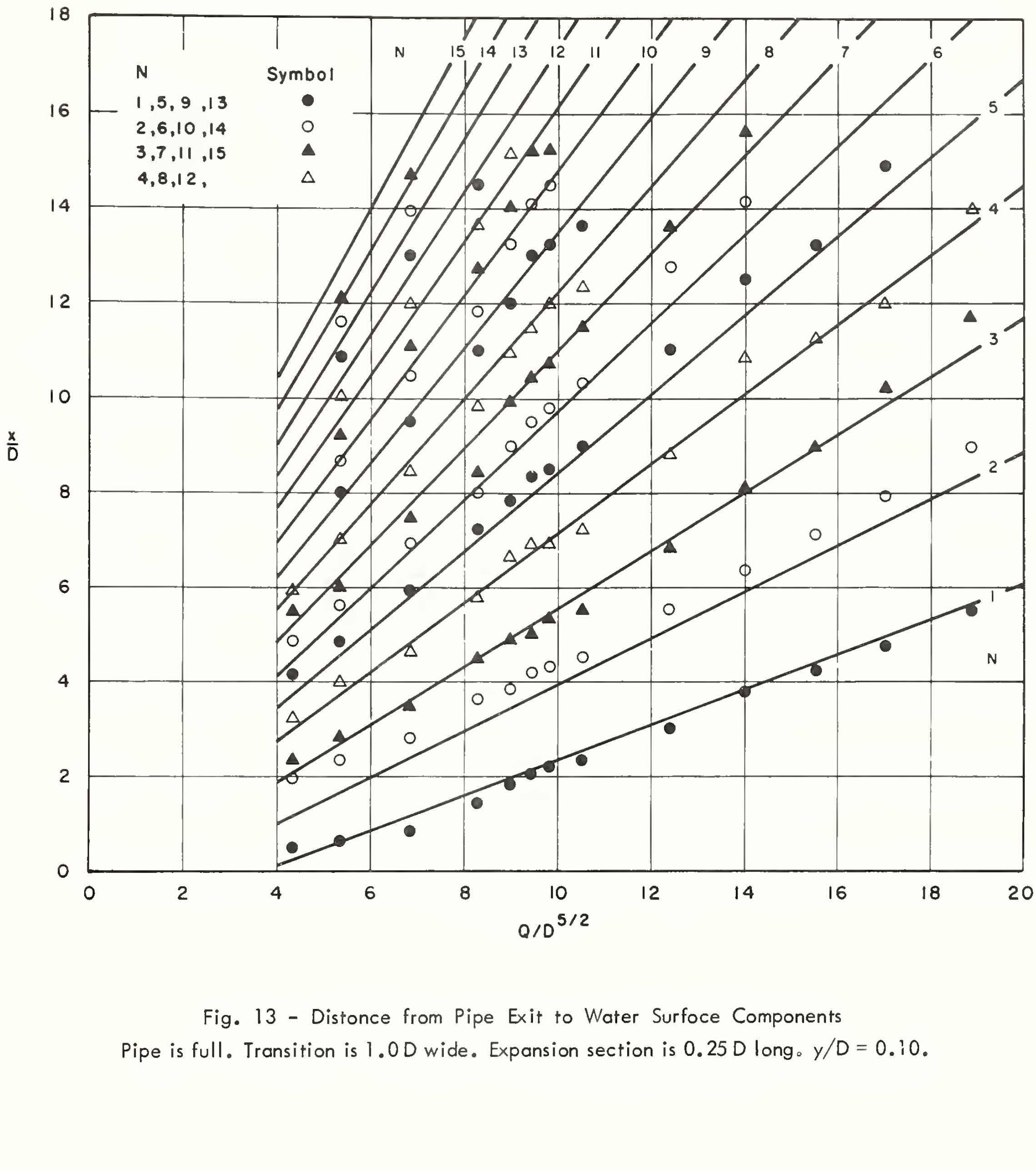


Fig. 13 - Distonce from Pipe Exit to Water Surfoce Components
Pipe is full. Transition is 1.0 D wide. Expansion section is 0.25 D long. y/D = 0.10.

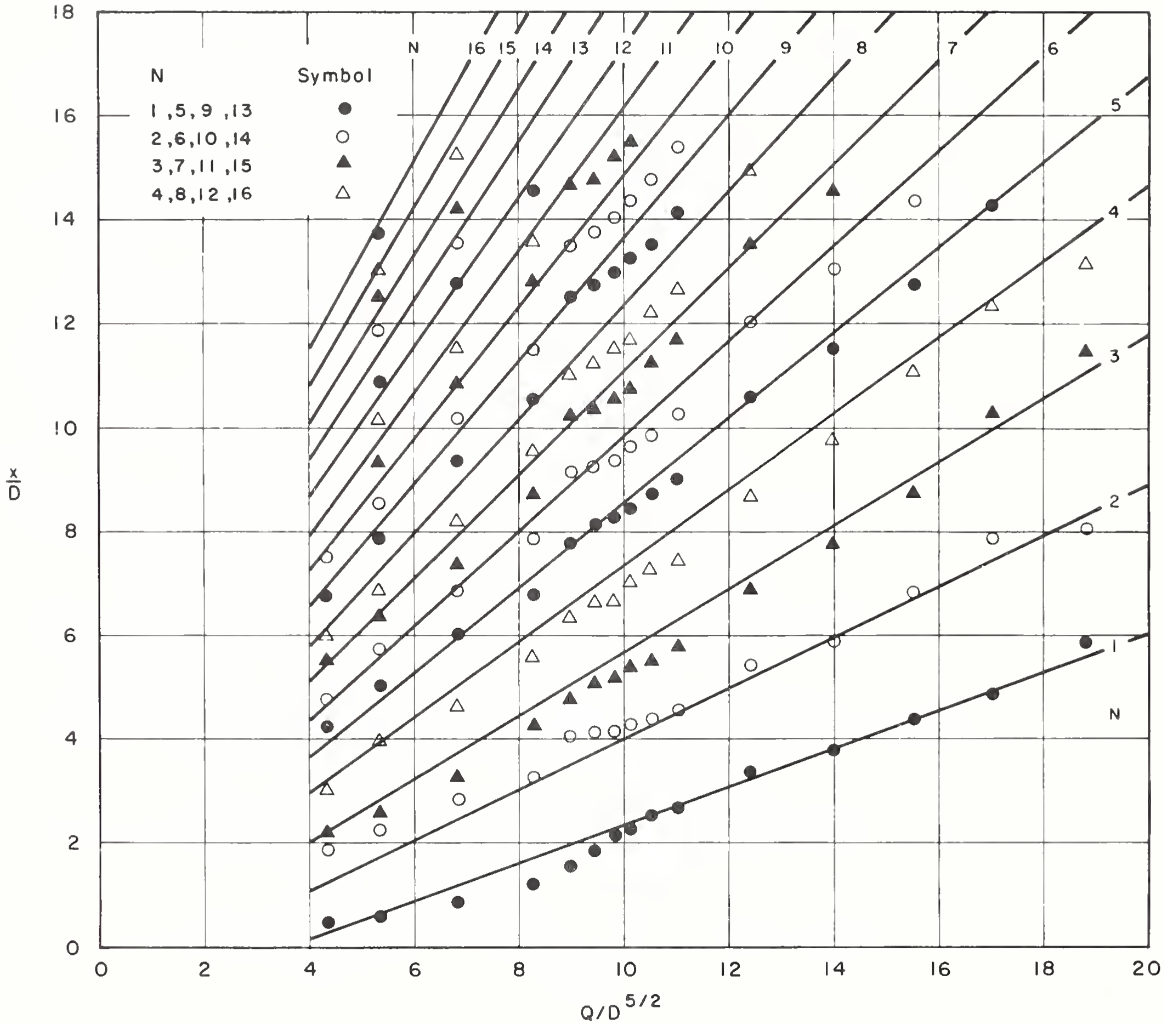


Fig. 14 - Distance from Pipe Exit to Water Surface Components

Pipe is full. Transition is 1.0 D wide. Expansion section is 0.25 D long. y/D = 0.15.

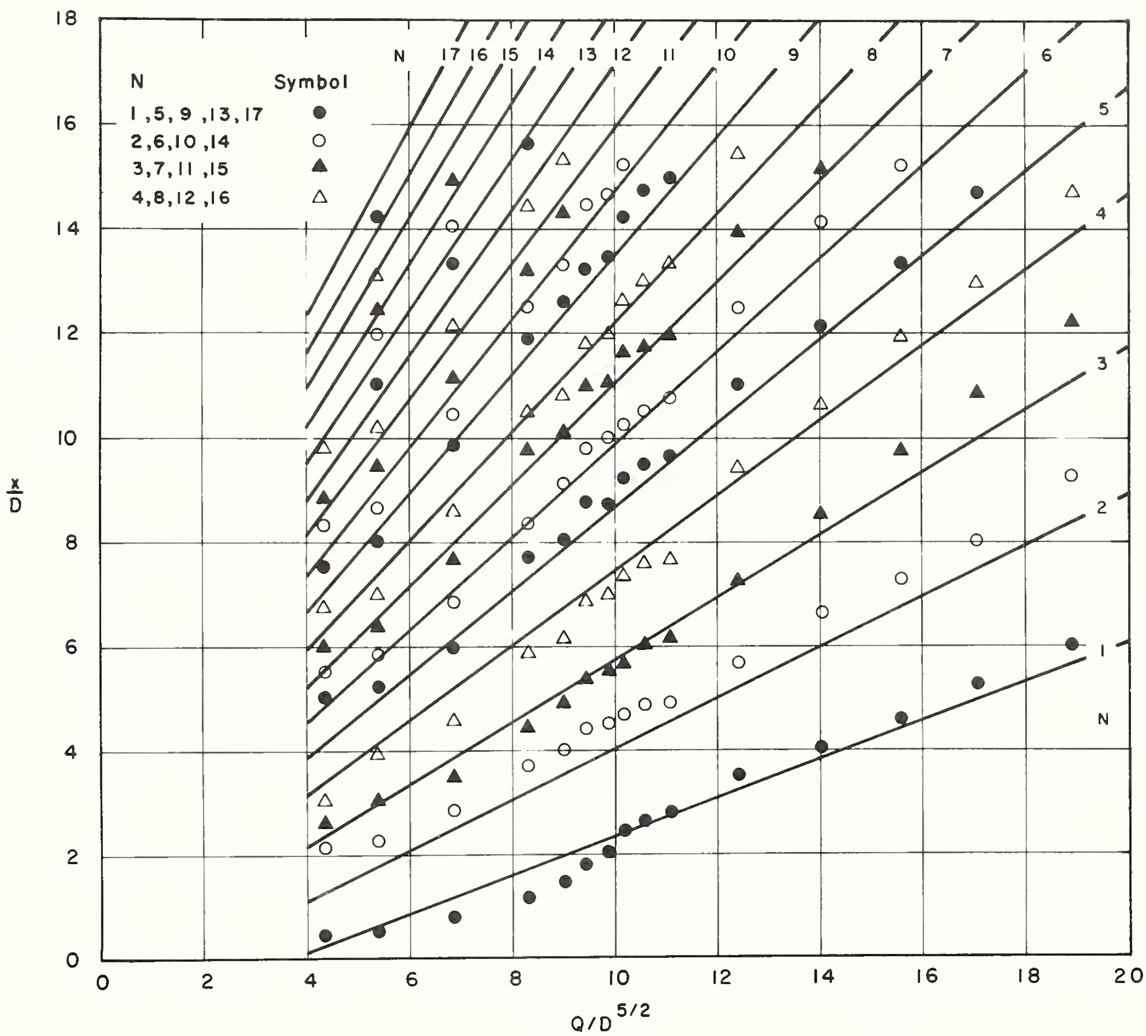


Fig. 15 - Distance from Pipe Exit to Water Surface Components
Pipe is full. Transition is 1.0 D wide. Expansion section is 0.25 D long. $y/D = 0.20$.

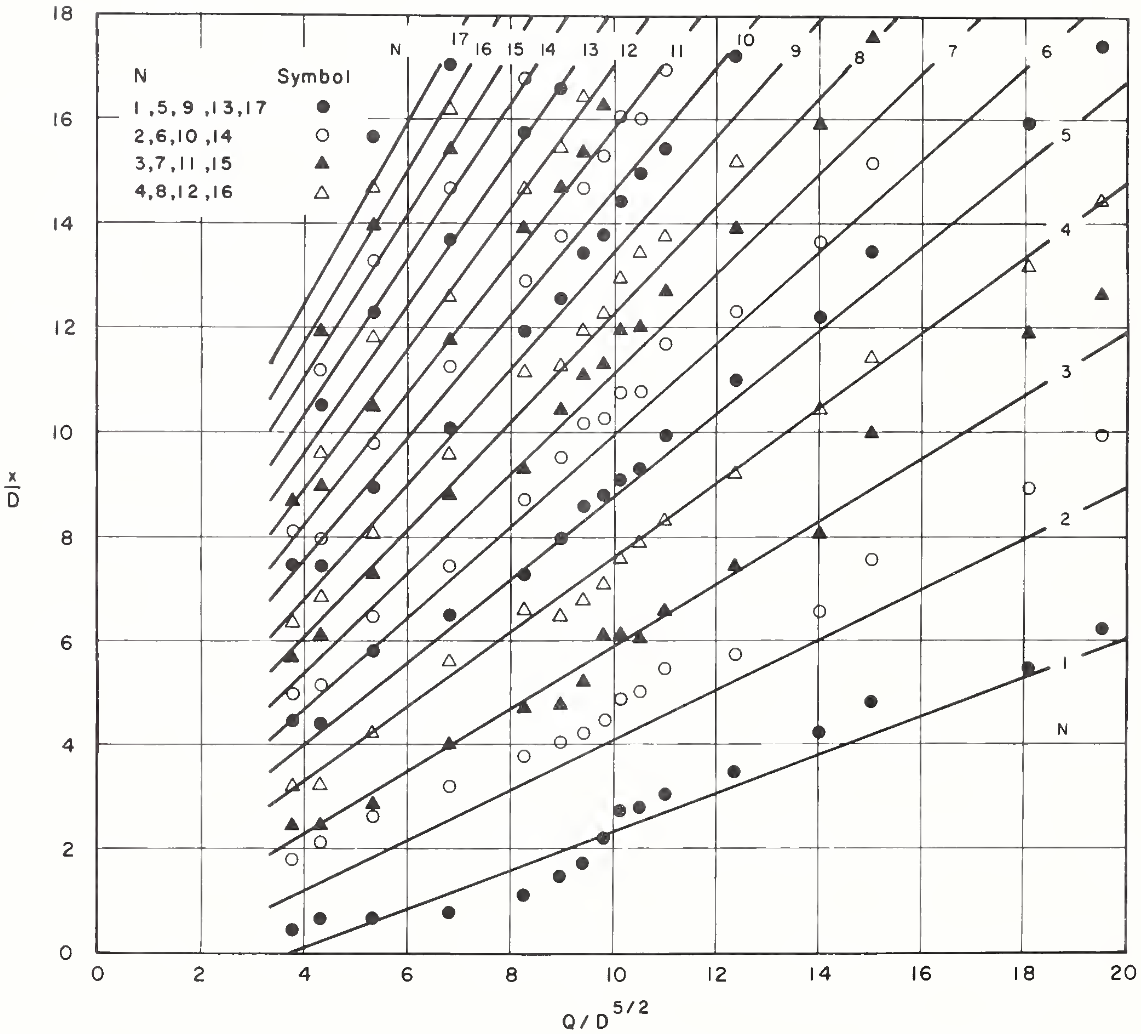


Fig. 16 - Distance from Pipe Exit to Water Surface Components
Pipe is full. Transition is 1.0 D wide. Expansion section is 0.25 D long. y/D = 0.25.

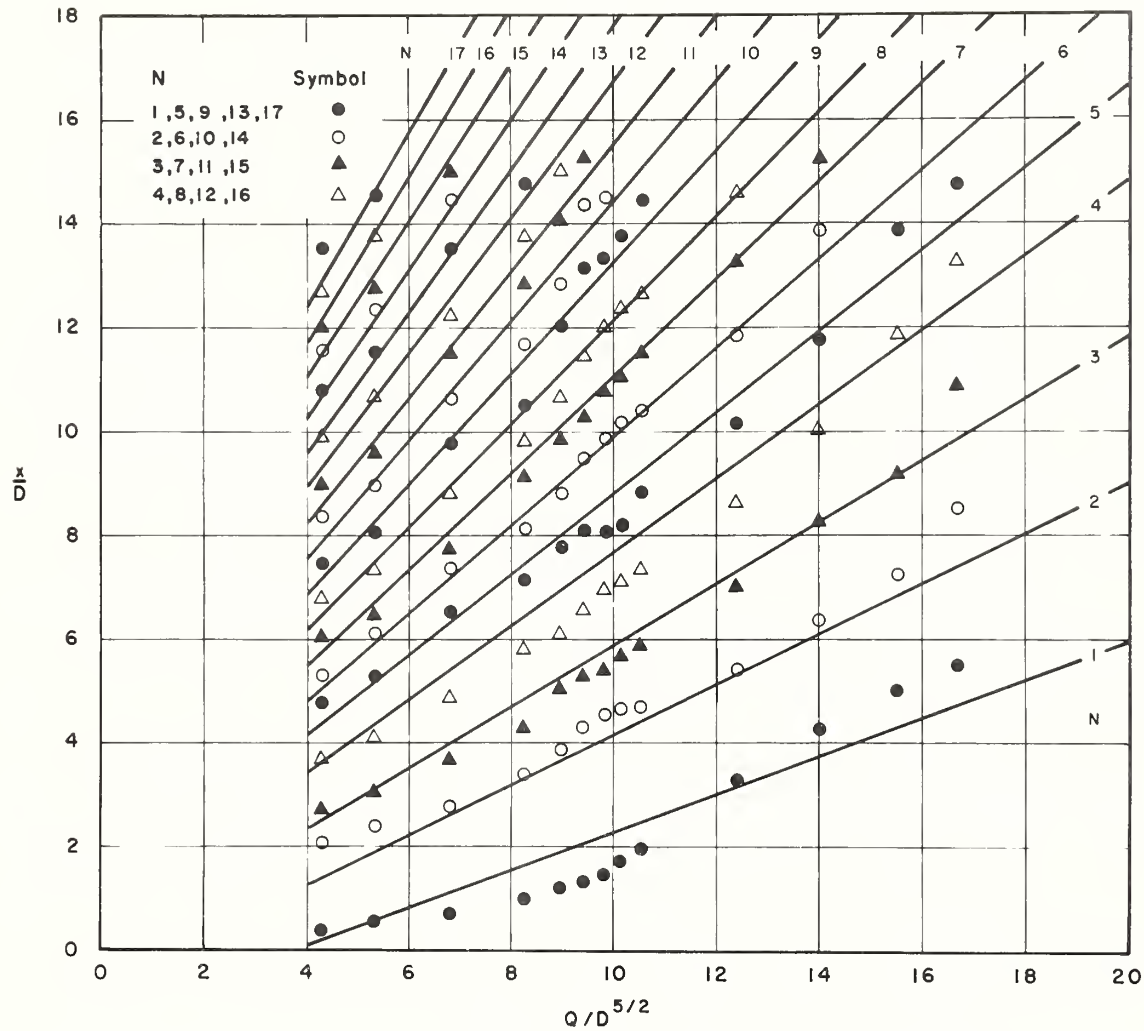


Fig. 17 - Distance from Pipe Exit to Water Surface Components
Pipe is full. Transition is 1.0 D wide. Expansion section is 0.25 D long. $y/D = 0.30$

(Continued fram page 12)

To obtain A, the slopes of the curves fitted to the data in Figs. 11 to 17 have been plotted against N in Fig. 18. Lines have been drawn in Fig. 18 to represent the slopes for each value of y/D. These lines have the equation

$$A = \frac{1}{32}\left[8 + \left(4 - \frac{y}{D}\right) N - 2.5 \langle N - 4 \rangle \sqrt{\frac{y}{D}}\right] \tag{7}$$

The quantity between the pointed brackets is zero for negative values.

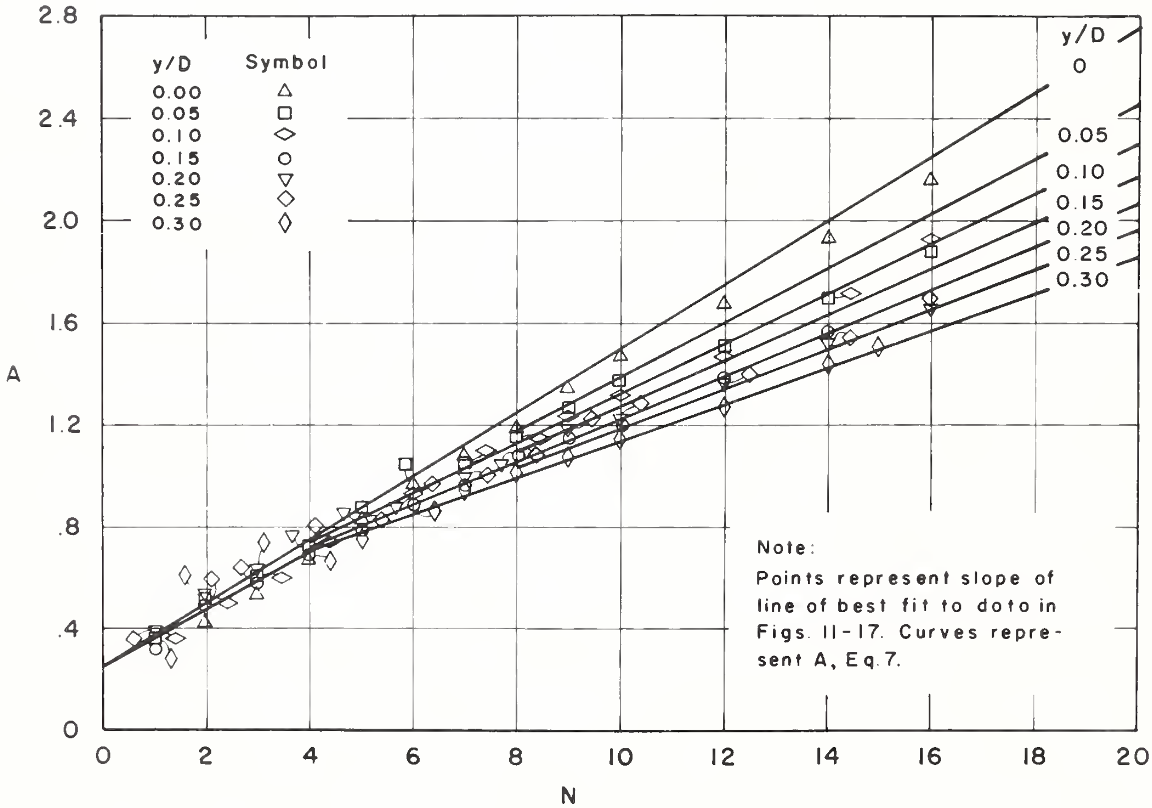


Fig. 18 - Values af A

It is anticipated that Eqs. 6 and 7 or Figs. 11, 12, 13, 14, 15, 16 and 17 will be used to determine the effect of the water surface elements and the transition channel length on the performance of a flip sill or other appurtenance. Eqs. 6 and 7 and Figs. 11 to 17 can also be used to determine the locations of the wave crests, the heights of which govern the sidewall height, or the locations of the other elements comprising the configuration of the water surface in the transition channel.

Wave Length

The wave length λ is taken as the distance between the crests of the same wave where it is reflected from the same side of the transition. This is illustrated in Fig. 19.

In terms of N and x relative to D,

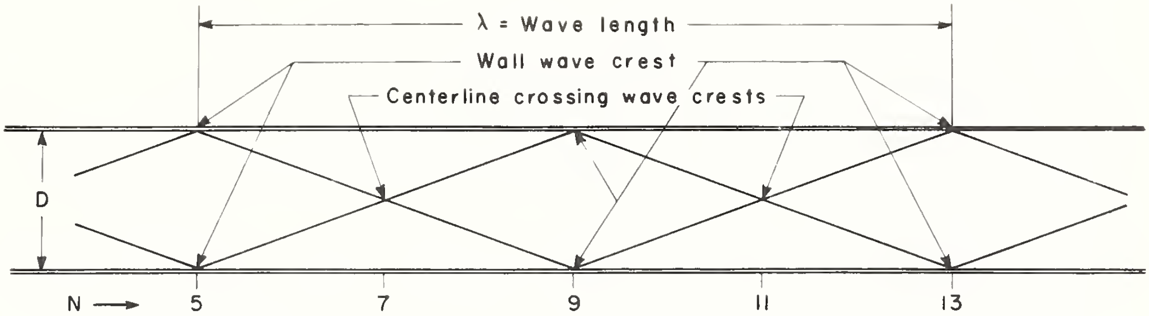


Fig. 19 - Wave Pattern in the Transition

$$\frac{\lambda}{D} = 8\,\frac{\Delta x/\Delta N}{D} \tag{8}$$

Eq. 8 can be evaluated by differentiating Eq. 6, which gives

$$\frac{\lambda}{D} = 8\,\frac{dx/dN}{D} = \frac{1}{4}\left(4 - \frac{y}{D} - 2.5\langle N - 4\rangle^{0}\sqrt{\frac{y}{D}}\right)\left(10\sqrt{\frac{y}{D}} + \frac{Q}{D^{5/2}}\right) \tag{9}$$

When $N > 4$, $\langle N - 4\rangle^{0} = 1$ and Eq. 9 becomes

$$\frac{\lambda}{D} = \frac{1}{4}\left(4 - \frac{y}{D} - 2.5\sqrt{\frac{y}{D}}\right)\left(10\sqrt{\frac{y}{D}} + \frac{Q}{D^{5/2}}\right) \tag{10}$$

For the special case of $y/D = 0$, Eqs. 9 and 10 reduce to

$$\frac{\lambda}{D} = \frac{Q}{D^{5/2}} \tag{10a}$$

All attempts to evaluate the wave lengths from the experimental data when $N \leq 5$ were unsuccessful. However, inspection of the data indicated that the observed wave lengths could be averaged for each $Q/D^{5/2}$ and y/D when $N > 5$. These averages are plotted in Fig. 20.

The curves for Eq. 10 drawn in Fig. 20 show that the agreement of Eq. 10 with the data is only fair and that the agreement decreases with increasing $Q/D^{5/2}$. A possible explanation for the poorer agreement at the higher value of $Q/D^{5/2}$ is that there are fewer wave crests in the test channel length so the averages are based on fewer measurements, the wave crests are long and flat making difficult the exact determination of the wave crest location, and the large amount of splash makes exact measurement impossible.

In spite of its lack of precision, Eq. 10 is presented because it probably represents a reasonable estimate of the wave length and the average distance between the water surface components when $N > 5$.

Depth of Flow at Nodes

The depth of flow measured at the nodes, where the transverse water surface is approximately horizontal, that is, at $N = 2, 4, 6, 8, \ldots$, is taken to be the average depth of flow from which the initial depth of flow and the energy losses in the transition can be determined. This information is needed to compute the average water surface profile in the transition.

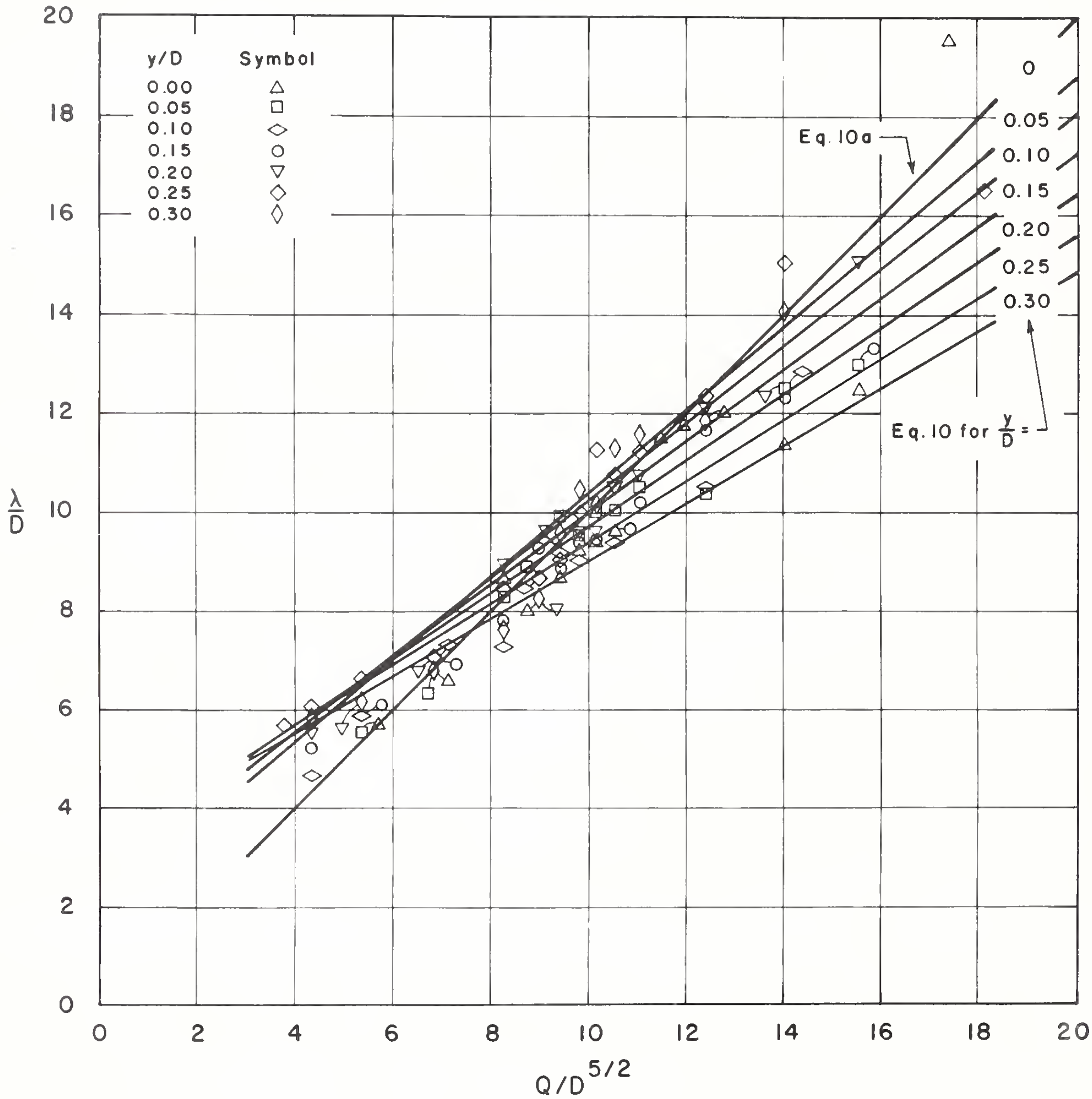


Fig. 20 - Wave Lengths for N > 5

Initial Depth of Flow.—The initial depth of flow is taken as the depth of flow d_2/D at N = 2. The experimental data are plotted in Fig. 21. The depths d_4/D at N = 4 are also plotted in Fig. 21 to provide additional data because of the difficulty of determining the elevation of the rough water surface. Because of the short distance and small losses between N = 2 and N = 4, d_4/D should approximate d_2/D. Examination of the data in Fig. 21 shows that sometimes d_2/D exceeded d_4/D, sometimes d_2/D was less than d_4/D, sometimes d_2/D was identical to d_4/D, and that no regular pattern is evident.

Equations have been derived that represent the data to within ±0.04D average and ±0.06D maximum deviation. These equations are:

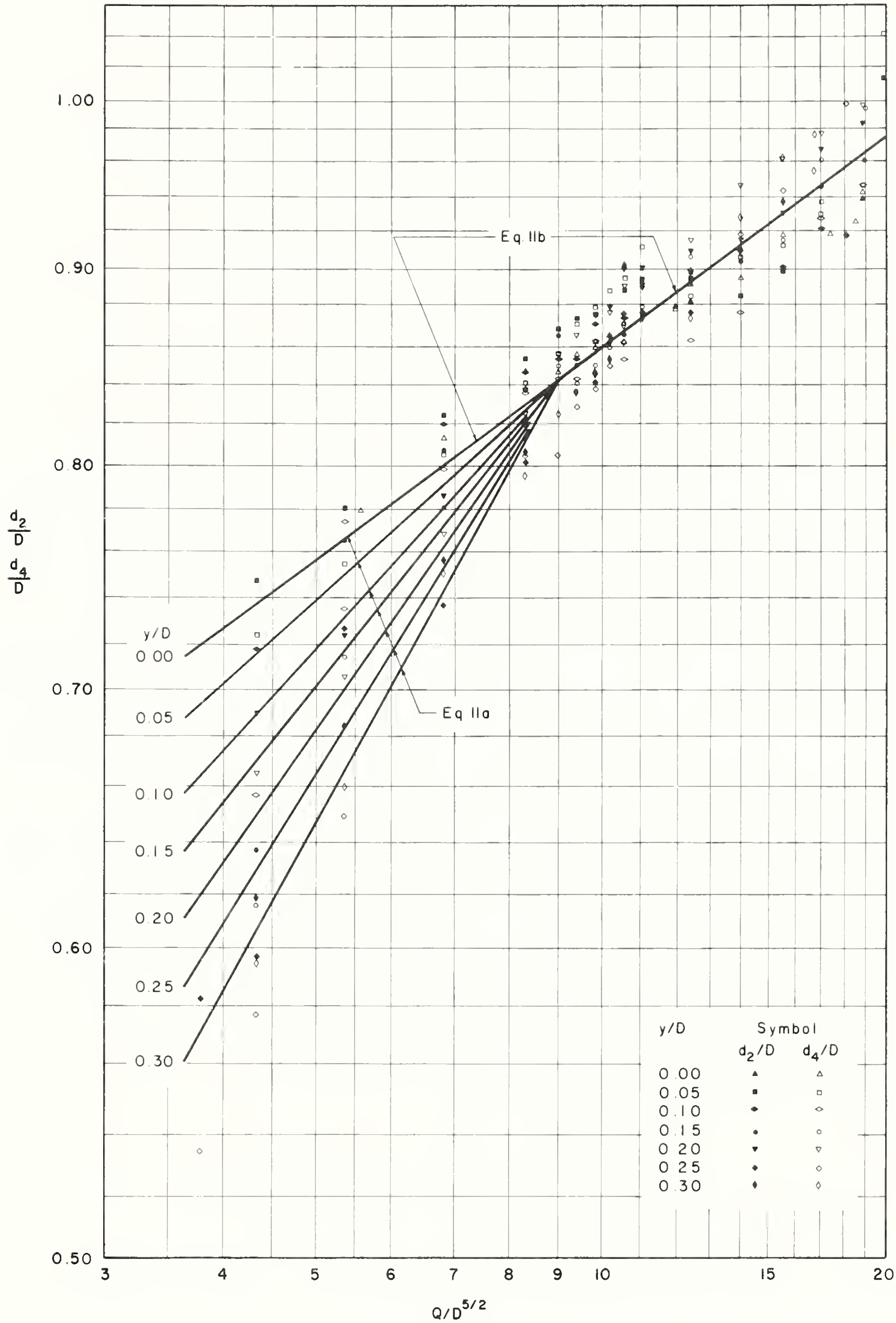


Fig. 21 - Initial Depth of Flow

for $\frac{Q}{D^{5/2}} \leq 9$,

$$\frac{d_2}{D} = \log^{-1}\left(\bar{1}.7542 - 0.840\frac{y}{D}\right)\left(\frac{Q}{D^{5/2}}\right)^{\left(0.180 + 0.88\frac{y}{D}\right)} \tag{11a}$$

and for $\frac{Q}{D^{5/2}} \geq 9$,

$$\frac{d_2}{D} = 0.5675\left(\frac{Q}{D^{5/2}}\right)^{0.180} \tag{11b}$$

Rate of Energy Loss.—The rate of energy loss along the transition channel was determined by computing the relative energy head at each node h_e/D and plotting this value against the relative distance along the transition x/D. A typical plot is shown in Fig. 22. The slope of the line fitted to the data represents the rate of energy loss h_ℓ/x. The rate of energy loss was determined for each experimental value of y/D and $Q/D^{5/2}$.

The data shown in Fig. 22 represent the approximate extremes of the discharges used during the tests. At low discharges there were 10 to 11 nodes—data points—to define the curve, and the surface was relatively smooth and therefore easy to measure accurately. As a result, the rate-of-energy-loss curve is well defined for low discharges. The definition of the curve decreases with increasing discharge. At high discharges there were only 2 or 3 nodes to define the curve, and the surface had so much splash that accurate measurement was impossible. For the higher discharges the rate of energy loss curve is poorly defined and its slope as determined is of low precision.

It was reasoned that the rate of total energy loss in the transition, like the rate of friction energy loss, would be a function of the velocity head. To verify this reasoning, the rate of energy loss h_ℓ/x was plotted against Q^2/D^5, a measure of the velocity head. The data are shown in Fig. 23. The resulting straight line is not well defined, especially at the high values of Q^2/D^5 where it was difficult to accurately evaluate h_ℓ/x. The curve drawn in Fig. 23 has the equation

$$\frac{h_\ell}{x} = 0.010 + 7 \times 10^{-5}\left(\frac{Q}{D^{5/2}}\right)^2 \tag{12}$$

or, in terms of the pipe diameter,

$$\frac{h_\ell}{D} = \left[0.010 + 7 \times 10^{-5}\left(\frac{Q}{D^{5/2}}\right)^2\right]\frac{x}{D} \tag{13}$$

Originally it had been reasoned that the rate of total energy loss in the transition channel could be divided into the rate of friction energy loss and a rate of turbulent energy loss, the turbulent energy loss being caused by the rough water surface and standing waves in the transition. However, when the friction

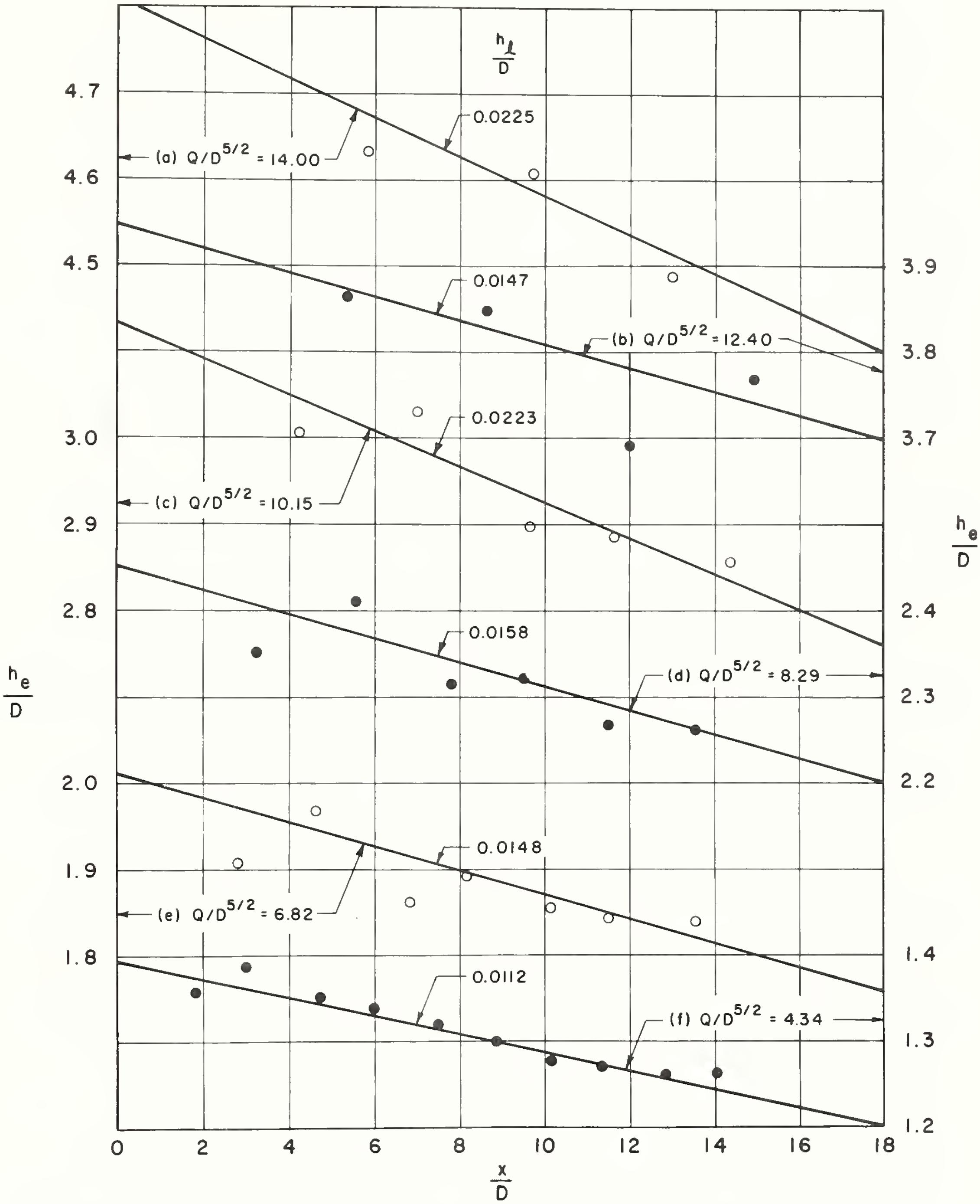


Fig. 22 - Typical Plot of Relative Energy versus Relative Distance Along the Transition

loss using an assumed Darcy-Weisbach friction factor f of 0.010 was subtracted from the total energy loss, the residual energy loss was negative for the higher discharges. This is, of course, impossible.

This anomaly was investigated by computing a psuedo Darcy-Weisbach friction factor f_e which represents the total energy loss in the transition. The value of f_e was computed at a point near the

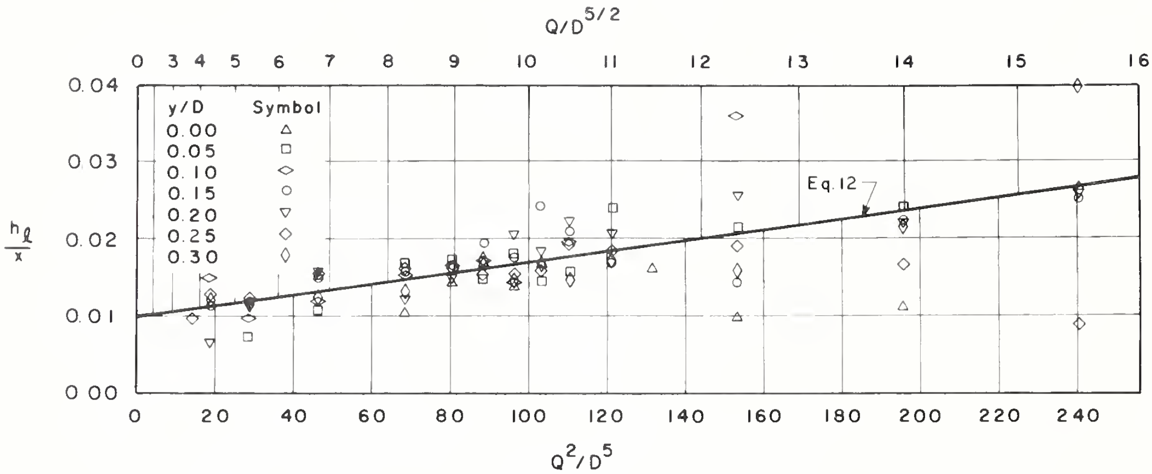


Fig. 23 - Rate of Energy Loss versus Relative Discharge

beginning of the transition channel where N = 2 and at the downstream end of the experimental transition channel where x = 18 ft. from the equation

$$f_e = \frac{h_\ell/x}{Q^2/D^5} \; \frac{(d/D)^3}{1 + 2d/D} \; 8g \tag{14}$$

where h_ℓ/x was computed from Eq. 12; $Q/D^{5/2}$ was assigned values of 4, 6, 8, 10, 12, 14 and 16; d_2/D

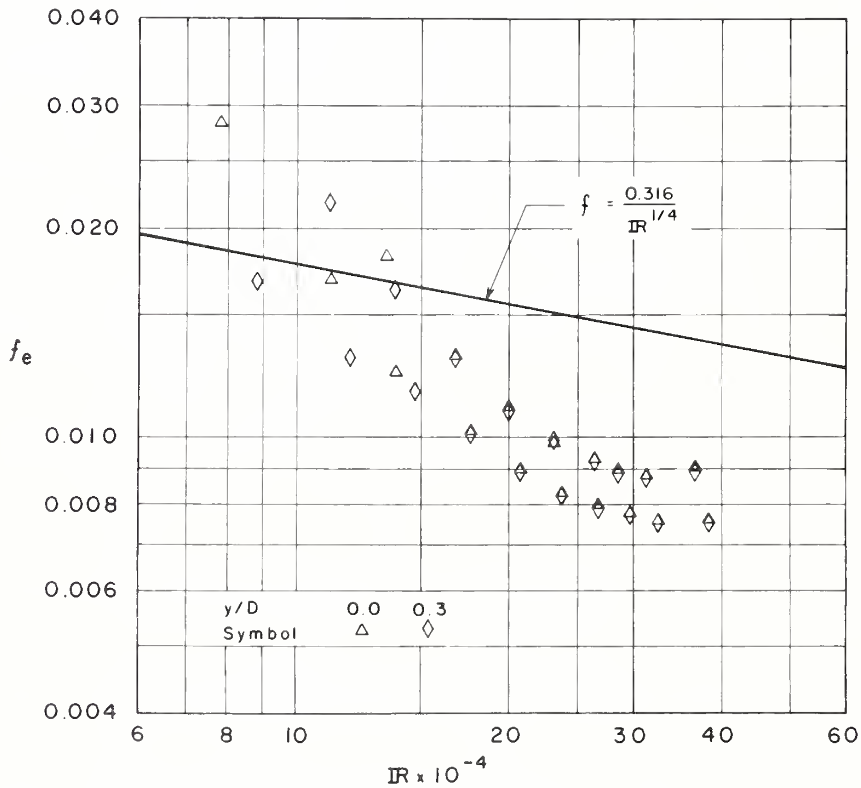


Fig. 24 - Energy Loss Factor versus Reynolds Number

at N = 2 was computed from Eqs. 11 for values of y/D of 0.0 and 0.3; d_N/D at x = 18 ft. was computed by substituting in the Bernoulli equation

$$\left(\frac{d_N}{D}\right)^3 - \left(\frac{Q^2/D^5}{2g(d_2/D)^2} + \frac{d_2}{D} - \frac{h_\ell}{D}\right)\left(\frac{d_N}{D}\right)^2 + \frac{Q^2/D^5}{2g} = 0 \qquad (15)$$

the values of d_2/D computed using Eqs. 11 and the rate of energy loss h_ℓ/D computed using Eq. 13.

The values of f_e computed from Eq. 14 are plotted in Fig. 24 against the Reynolds number $\mathbb{R}$, where

$$\mathbb{R} = \frac{Q/D^{5/2}}{1 + 2d/D} \; \frac{4D^{3/2}}{\nu} \qquad (16)$$

For comparison, the Blasius curve for the friction factor f for turbulent flow insmooth pipes

$$f = \frac{0.316}{\mathbb{R}^{1/4}} \qquad (17)$$

is also shown in Fig. 24.

It can be seen that most of the energy loss factors computed from Eq. 14 fall below the friction factor curve for smooth pipe. The authors can offer no explanation for this anomaly.

For lack of a more satisfactory procedure, it is suggested that Eqs. 12 or 13 be used to compute the rate of energy loss along the transition channel.

Wave Heights at Sidewalls

The first (N = 1) wall wave height varied differently with the discharge and transition floor elevation than did the wall waves for N = 5, 9, 13, . . . so the test results for the first wall wave height will be presented separately.

First Wall Wave Height.—The relative heights of the first sidewall wave d_1/D for each y/D tested are presented in Fig. 25.

It can be seen in Fig. 25 that the relative height of the first wall wave varies irregularly with the discharge. The first wall wave increases in height with the discharge until $Q/D^{5/2}$ is between 6 and 7, decreases until $Q/D^{5/2}$ is between 9 and 11, and subsequently increases. The reason is apparently due to the manner in which the water leaves the pipe. A possible explanation is that at low discharges the jet drops onto the transition floor and traps water under it and in the expansion section. At high discharges the jet "aspirates" the water from upstream of its point of impingement on the transition floor. The combined effect produces a minimum height at a $Q/D^{5/2}$ between 9 and 11. Another possibility is the relative effect of the falling jet on the rising disturbance wave and the variation of this effect as the discharge changes.

An equation has been developed that can be used to estimate the maximum height of the first sidewall wave. It is

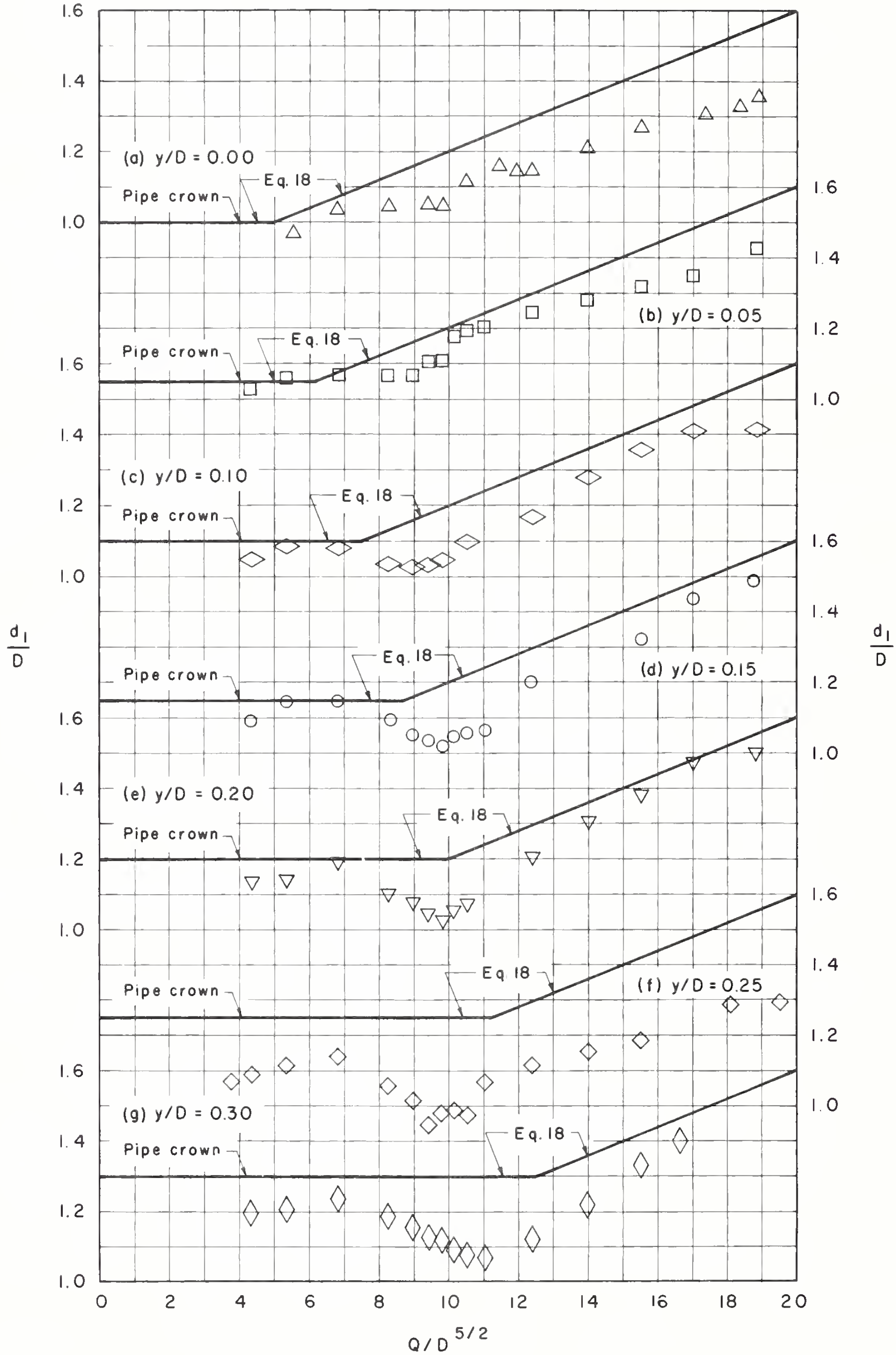


Fig. 25 - Relative Height of First Sidewall Wave

$$\frac{d_1}{D} = 0.8 + 0.04 \frac{Q}{D^{5/2}} \geq 1 + \frac{y}{D} \qquad (18)$$

Eq. 18 is shown in Fig. 25. It is an envelope curve such that all observed wall wave heights fall on or below the curve.

The agreement of Eq. 18 with the data is highly variable for reasons that are not wholly apparent. Some of the variation is due to difficulties in obtaining precise measurements while other variations, especially in Fig. 25f, suggest that the observer may have unintentionally used different criteria when determining the wave heights. The precision of the results is not high, the equation should not be extrapolated beyond the range of the experiments, and the designer should use judgment in the application of the equation.

Eq. 18 is in two parts. The first part gives the sidewall wave height when $Q/D^{5/2}$ exceeds 5 to 12.5, depending on how far the transition floor is below the pipe invert. The second part of Eq. 18 gives the minimum sidewall wave height at lesser values of $Q/D^{5/2}$.

It might seem that the sidewall height based on the wall wave height could be decreased for some transition floor levels at some relative discharges $Q/D^{5/2}$ between about 7 to about 12. However, for the pipe to discharge these higher flows, the flow must first pass through a discharge producing a higher wall wave. Therefore, for sidewall height design purposes, the minimum wall wave height for use at the lower flows is taken to be the maximum wall wave height observed for these lower flows. This is not a severe restriction because the minimum wall wave height and presumably the minimum sidewall height is at the elevation of the crown of the pipe, as is shown in Fig. 25.

Succeeding Wall Wave Heights.—Succeeding ($N = 5, 9, 13, \ldots$) wall wave heights are dependent on the wall wave height immediately upstream and the intervening dissipation of energy. The succeeding wall wave heights should decrease with distance, although the experimental data do not conclusively verify this reasoning.

The heights of the succeeding wall waves are measured above the average depth of flow computed from Eq. 15. In other words, the average relative depth of flow d_{aN}/D computed by substituting d_{aN} for d_N in Eq. 15 was subtracted from the measured relative wall wave height d_N/D to obtain the relative wall wave height above the computed average flow depth d_{wN}/D. The relative heights d_{wN}/D are plotted against $Q/D^{5/2}$ in Fig. 26 for $N = 5$, 9, 13 and 17.

Envelope curves have been drawn in Fig. 26 to give the maximum height of the sidewall waves (and the minimum sidewall height) above the average depth of flow. For wall wave 5,

$$\frac{d_{w5}}{D} = 0.015 \frac{Q}{D^{5/2}} \qquad (19)$$

and for wall waves 9, 13 and 17,

$$\frac{d_{w9}}{D} = \frac{d_{w13}}{D} = \frac{d_{w17}}{D} = 0.010 \frac{Q}{D^{5/2}} \qquad (20)$$

Although some wall wave heights at the high discharges fall below the heights at intermediate discharges, the envelope curve cannot be lowered for high design discharges because the flow, on a rising hydrograph, must pass through a low discharge to reach the higher design discharge.

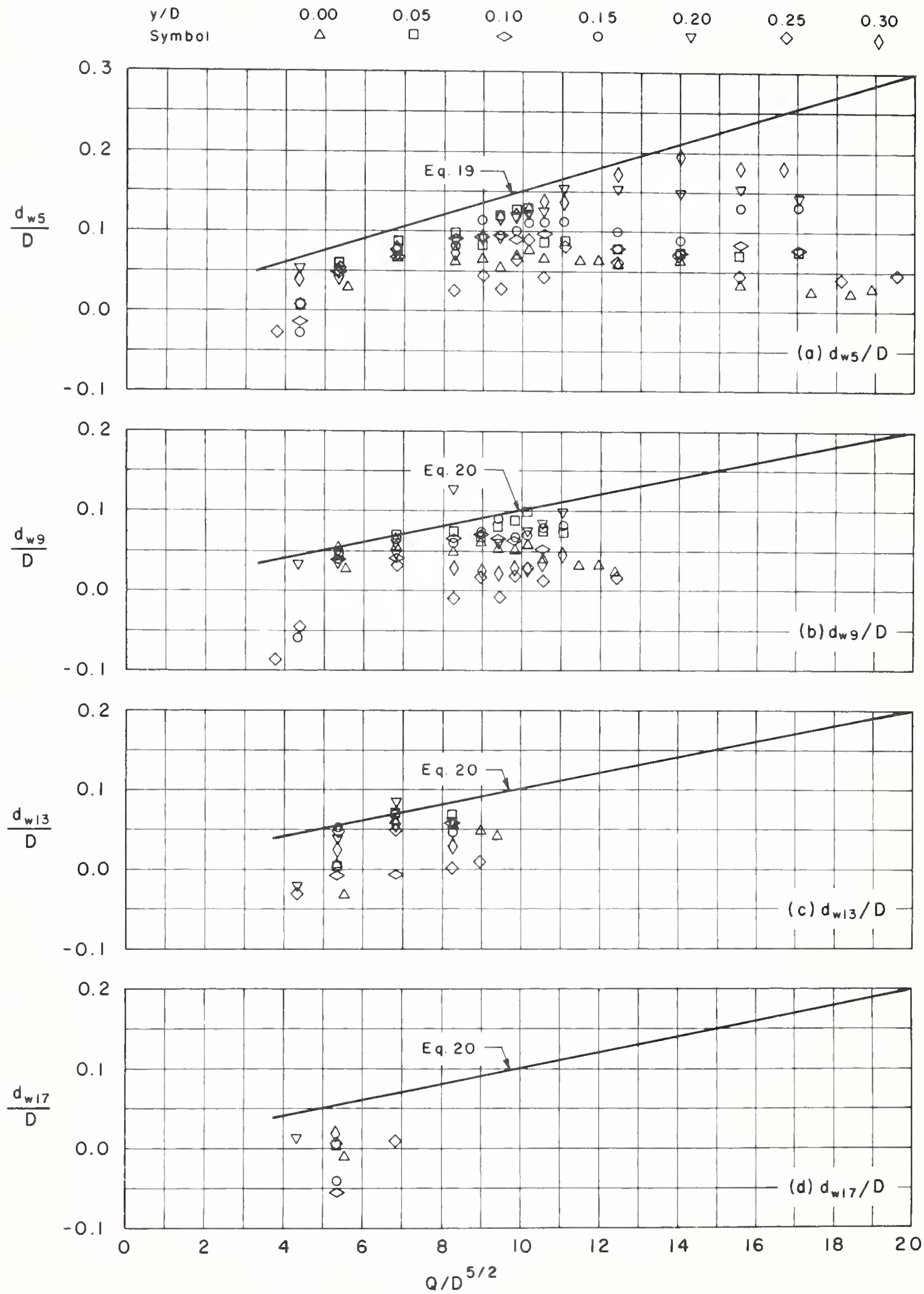


Fig. 26 - Height of Wall Waves 5, 9, 13 and 17 Above the Average Depth of Flow

The wall wave heights at N = 17 fall well below the Eq. 20 curve. However, because the few data obtained for N = 17 poorly define the curve, the additional safety provided by Eq. 20 is warranted. Moreover, since wall wave 17 is near the downstream end of the transition channel, the extra length of higher sidewall required will be small. The additional safety will therefore not be costly.

Wave Heights at Centerline

As noted in the section entitled "Description of Flow" and illustrated at N = 3, 7 and 11 in Fig. 5, the waves reflected from opposite sidewalls are augmented in height where they cross the transition channel centerline.

Because its usefulness is not apparent, an equation for the centerline wave height has not been developed. However, the data obtained are presented in Fig. 27. The centerline wave heights for N = 3, 7, 11 and 15 shown in Fig. 27 are measured above the average depth of flow as computed from Eq. 15.

Agreement of Equations with Experimental Data

The equations developed from the experimental data were compared with the experimental data to see how representative they are. The results are presented in Table 2, appendix, and are discussed in the following sections.

Distance from Pipe Exit to Surface Elements

The distance from the pipe exit to the surface elements was computed using Eqs. 6 and 7. The results are listed in Table 2, appendix, under x/D as the EQ. value. The experimental or TEST values of x/D have been subtracted from the EQ. values and the differences listed in the DIFF. column. A positive difference indicates that the equation locates the surface element downstream of the observed location; a negative difference indicates that the equation locates the surface element upstream of the observed location.

Eqs. 6 and 7 are least representative of the data for y/D = 0.00. For y/D = 0.00, the maximum difference is a downstream (+) difference of 2.20 D (for y/D = 0.00 and $Q/D^{5/2}$ = 15.52) and the average difference is +0.68 D. For all other values of y/D the equations give a maximum downstream difference of +1.43 D (for y/D = 0.10 and $Q/D^{5/2}$ = 8.29), a maximum upstream difference of - 1.30 D (for y/D = 0.25 and $Q/D^{5/2}$ = 19.54), and an average downstream difference of +0.11 D from the observed locations of the surface elements.

It thus appears that Eqs. 6 and 7 will locate the positions of the surface elements within a maximum range of ±1.4 pipe diameters of the observed location, except possibly for wall wave 1 when y/D = 0.00. The average difference will be about +0.11 pipe diameters. Some of these differences can be attributed to difficulties in making the measurements; possibly the equations represent better values of x/D than is indicated by the magnitude of the tabulated differences.

Wall Wave Height

The maximum heights of the sidewall waves were computed using Eqs. 18, 19 and 20. The results are listed in Table 2, appendix, under d/D as the EQ. values for odd values of N. The differences between the TEST and the EQ. values of d/D are listed in the first DIFF. column.

The equations represent envelope curves which give maximum values of d/D. Therefore all differences between the EQ. and TEST values of d/D should be positive.

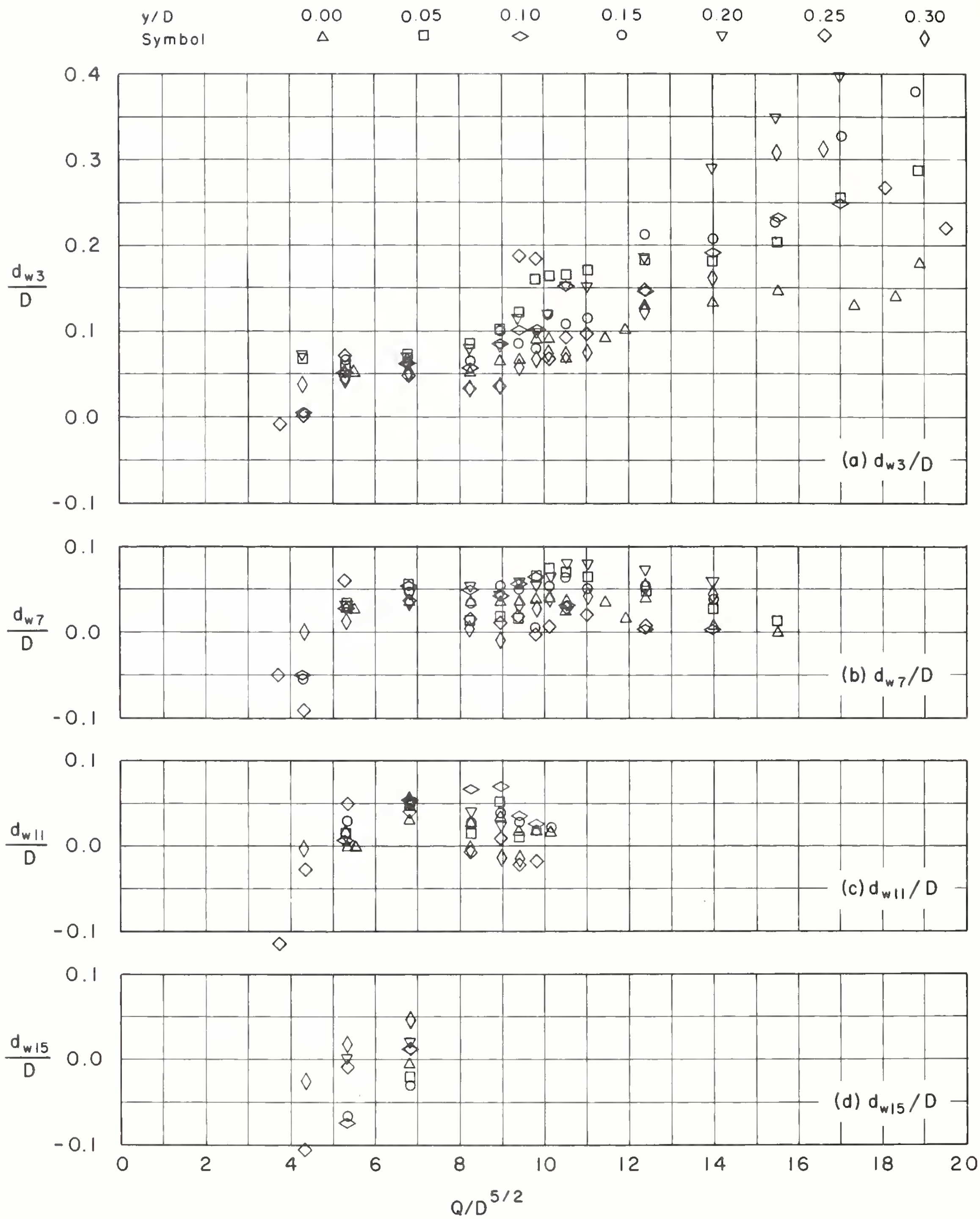


Fig. 27 - Height of Centerline Waves 3, 7, 11 and 15 Above the Average Depth of Flow

If Eqs. 18, 19 and 20 are used to give the sidewall height, negative values of d/D would indicate that the wall wave overtopped the sidewall. Only 6 of 305 observed sidewall wave heights were negative: four by less than - 0.01 D, one by -0.02 D and one by - 0.04 D. These are small negative values, they are few in number, and they may possibly be the result of splash which made it impossible to secure accurate measurements.

The maximum positive difference or freeboard above the crest of the sidewall waves is +0.31 D. The average difference between the EQ. and TEST values of the sidewall wave height or average freeboard is +0.08 D.

Eqs. 18, 19 and 20 can be used to estimate the minimum heights of the sidewall for the transition channel in order to prevent overtopping the sidewall. Use of these equations will provide a minimum freeboard of about zero, a maximum freeboard of about 0.3 D, and an average freeboard above the wall wave crests of about 0.08 D. Since spray can be expected above the heights given by Eqs. 18, 19 and 20, the designer may wish to provide additional freeboard.

Depth of Flow at the Nodes

The depth of flow at the nodes was computed using Eqs. 15, 11, 13, 6 and 7. The results are listed in Table 2, appendix, under d/D as the EQ. values for even values of N. The TEST values have been subtracted from the EQ. values and the differences listed in the second DIFF. column.

The equations represent mean values of d/D at the nodes. The maximum and minimum differences between the computed and test values of d/D at the nodes are +0.13 and - 0.06. The maximum positive difference occurred at the lowest discharge for each y/D. The average difference is +0.01 D, indicating that Eqs. 15, 11, 13, 6 and 7 well represent the experimental data.

COMPUTATION OF SURFACE ELEMENT LOCATIONS AND DEPTHS

To facilitate the use of the test results by the designer, the computations of the surface element locations and heights are summarized and explained in detail.

Distance from Pipe Exit to Surface Elements

The distance from the pipe exit to the locations of the surface elements is computed from Eq. 6

$$\frac{x_N}{D} = A\left(\frac{Q}{D^{5/2}} + 10\sqrt{\frac{y}{D}}\right) - 4.5\sqrt[4]{\frac{y}{D}} \tag{6}$$

where

$$A = \frac{1}{32}\left[8 + \left(4 - \frac{y}{D}\right)N - 2.5\langle N - 4\rangle\sqrt{\frac{y}{D}}\right] \tag{7}$$

x_N = the distance from the pipe exit to surface element N, in feet

D = the pipe diameter, in feet

Q = the rate of flow, in cubic feet per second

y = the vertical distance between the floor of the transition channel and the pipe invert, in feet

N = the surface element or quarter wave length number

$\langle\ \rangle$ = indicates that the quantity between the pointed brackets is zero for negative numbers

The computation steps are:

1. Select the pipe diameter D, the discharge Q, the vertical distance between the floor of the transition channel and the pipe invert y, and the element number N.
2. Compute $Q/D^{5/2}$ and y/D
3. Use Eq. 7 to compute A
4. Use Eq. 6 to compute x_N/D
5. Multiply x_N/D by D to obtain x_N

The resulting value of x_N should locate surface element N within about ±1.4 pipe diameters of the correct position, the average error being about 0.11 pipe diameters.

Correction for Pipe Slope

The pipe was horizontal for the tests. However, the distance from the pipe exit to the surface elements x_N/D can be corrected for the pipe slope S by subtracting from x_N/D the distance from the pipe exit to the point where the slope of the trajectory of the initially horizontal jet equals the pipe slope. The jet trajectory is given by Eq. 4. When the differential of Eq. 4 is solved for the slope of the jet trajectory and equated to the pipe slope,

$$\frac{d(y/D)}{d(x/D)} = \frac{g\pi^2}{16\left(Q/D^{5/2}\right)^2}\frac{x}{D} = \tan\alpha = \tan\left(\sin^{-1} S\right)$$

The correction to x_N/D for pipe slope to be subtracted from x_N/D is

$$\frac{x}{D} = \frac{16}{g\pi^2}\tan\alpha\left(\frac{Q}{D^{5/2}}\right)^2$$

Depth of Flow

The depth of flow in the transition channel is determined at an initial point which is then used to compute the depths of flow at the nodes or the average depth of flow at any location in the transition channel.

Initial Depth of Flow

The initial depth of flow is located at the first node where N = 2. It is computed from Eqs. 11, where for $\frac{Q}{D^{5/2}} \leq 9$,

$$\frac{d_2}{D} = \log^{-1}\left(\bar{1}.7542 - 0.840\frac{y}{D}\right)\left(\frac{Q}{D^{5/2}}\right)^{\left(0.180 + 0.88\frac{y}{D}\right)} \tag{11a}$$

and for $\frac{Q}{D^{5/2}} \geq 9$,

$$\frac{d_2}{D} = 0.5675\left(\frac{Q}{D^{5/2}}\right)^{0.180} \tag{11b}$$

where d_2 = the depth of flow at the surface element when N = 2, the first node, in feet

The computation steps are:

1. Select D, Q and y as before. N = 2
2. Compute $Q/D^{5/2}$ and y/D as before
3. If $Q/D^{5/2} \leq 9$, use Eq. 11a to compute d_2/D;
   if $Q/D^{5/2} \geq 9$, use Eq. 11b to compute d_2/D
4. Multiply d_2/D by D to obtain d_2

The resulting values of d_2 should be within ±0.04 pipe diameters but may be as large as ±0.06 pipe diameters.

Depth of Flow at Succeeding Nodes

The depth of flow at the succeeding nodes, where N = 4, 6, 8, 10, 12, . . . , is computed from Eq. 15

$$\left(\frac{d_N}{D}\right)^3 - \left(\frac{Q^2/D^5}{2g(d_2/D)^2} + \frac{d_2}{D} - \frac{h_\ell}{D}\right)\left(\frac{d_N}{D}\right)^2 + \frac{Q^2/D^5}{2g} = 0 \tag{15}$$

where d_N = the depth of flow at surface element N, in feet
h_ℓ = the energy loss in the transition channel, in feet

The computation steps are:

1. Select D, Q and N as for computing the distance to the surface elements
2. Compute $Q^2/D^5 = (Q/D^{5/2})^2$
3. Compute the relative depth of flow at the first node d_2/D, using the procedure for computing the initial depth of flow
4. Compute the relative distance to nodes 2 and N, x_2/D and x_N/D, using the procedure for computing the distance to the surface elements
5. Use Eq. 13

$$\frac{h_\ell}{D} = \left[0.010 + 7 \times 10^{-5}\left(\frac{Q}{D^{5/2}}\right)^2\right]\frac{x_\Delta}{D} \tag{13}$$

   to compute the relative loss of energy between x_2/D and x_N/D. In Eq. 13, x_Δ = the distance, in feet, between x_2 and x_N and $x_\Delta/D = x_N/D - x_2/D$
6. Use Eq. 15 to compute d_N/D. There are three values of d_N/D. The negative root is imaginary and be discarded. The other two roots are positive and represent the conjugate depths for the hydraulic jump. The smaller positive root gives the correct value of d_N/D.
7. Multiply d_N/D by D to obtain the depth of flow at node N, d_N

Average Depth of Flow

Because the sidewall wave height is measured above the average depth of flow, it is necessary to compute the average depth of flow d_{aN} at the wall wave locations, where N = 1, 5, 9, ... The computation procedure is identical to that described for computing the depth of flow at succeeding nodes, except that N = 1, 5, 9, 13, . . . and d_{aN} is substituted for d_N.

If the average depth of flow is needed at other locations in the transition channel, this depth can be computed using the procedure described for computing the depth of flow at succeeding nodes, except that in step 4 the distance x_N/D becomes x/D, x_N/D is selected rather than computed, and x_N/D is not necessarily the distance to a surface element.

The computed depth of flow at the nodes and the average depth of flow should be within the range 0.13 D above (greater than) to 0.06 D below (less than) the actual depth of flow. The average difference between the computed and actual flow depths should be about 0.01 D.

Wall Wave Heights

The wall wave heights determine the minimum height of the transition channel sidewalls. The freeboard above the computed minimum wall wave height is left to the judgment of the designer.

Different procedures are used for computing the first and succeeding sidewall wave heights. Sidewall waves occur at N = 1, 5, 9, 13, ...

First Wall Wave Height

The maximum height of the first sidewall wave (N = 1) is given by Eq. 18

$$\frac{d_1}{D} = 0.8 + 0.04 \frac{Q}{D^{5/2}} \geq 1 + \frac{y}{D} \tag{18}$$

where d_1 = the height of the first sidewall wave.

The computation steps are:

1. Select D, Q, y and N
2. Compute $Q/D^{5/2}$ and y/D
3. Use Eq. 18 to compute d_1/D
4. Multiply d_1/D by D to obtain d_1

The resulting computed value of d_1 should range from 0.00 D to 0.31 D above the actual first wall wave crest, the average freeboard being about 0.12 D.

Succeeding Wall Wave Heights

The maximum heights for the succeeding (N = 5, 9, 13 and 17) wall waves above the average depth of flow are given by Eqs. 19 and 20.

$$\frac{d_{w5}}{D} = 0.015 \frac{Q}{D^{5/2}} \tag{19}$$

and

$$\frac{d_{w9}}{D} = \frac{d_{w13}}{D} = \frac{d_{w17}}{D} = 0.010 \frac{Q}{D^{5/2}} \tag{20}$$

where d_{wN} = the wave height above the computed average depth of flow, in feet

The computation steps are:

1. Select D, Q, y and N
2. Compute $Q/D^{5/2}$ and y/D
3. Use the procedure for computing the average depth of flow to compute the average flow depth at N = 5, 9, 13 or 17
4. Use Eqs. 19 or 20 to compute d_{wN}/D
5. Multiply d_{wN}/D by D to obtain d_{wN}
6. Add the depths obtained in steps 3 and 5 to determine d_N, the maximum height of wall wave N, in feet

The resulting computed value of d_N should be from 0.13 D above to 0.04 D below the actual wall wave crest, the average freeboard being about 0.06 D.

Centerline Wave Heights

If the height of the centerline waves is desired, the computation procedure is the same as described under Succeeding Wall Wave Heights except that N = 3, 7, 11 and 15 and the height of the centerline wave above the average depth of flow is taken from Fig. 27.

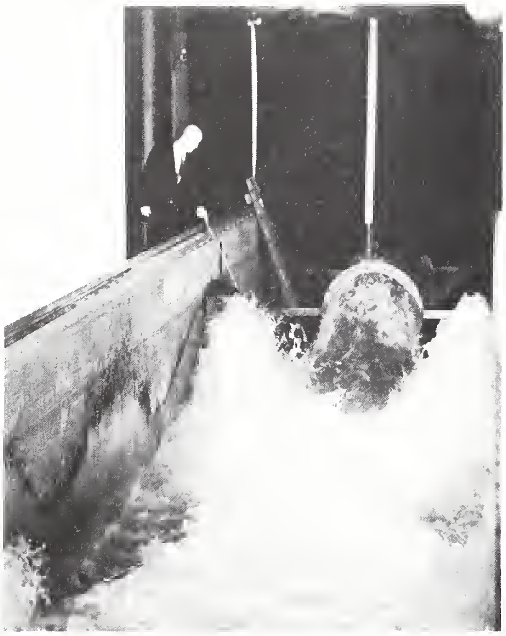
PROTOTYPE-MODEL COMPARISONS

An opportunity to make prototype-model comparisons presented itself after the previously reported tests and analyses had been completed and the report had been drafted. Advantage was taken of this opportunity. The comparisons consisted of the wave locations and heights for two discharges and a visual check on the spread of the jet as it passed through the pipe expansion section.

The prototype was a 3.00-ft. diameter pipe installed with its invert 0.63 ft. (0.21 D) above the floor of the 9-ft. (actually 3.01 D) wide main test channel. No model tests had been made on a channel this wide, so the model channel was adjusted to reproduce the prototype channel. The model pipe was 4 in. in diameter, so the prototype-model scale ratio is 9:1. Because of difficulties in controlling the flow rate in the prototype, only two discharges were used for the comparisons. The prototype discharges were 196 cfs ($Q/D^{5/2}$ = 12.6) and 141 cfs ($Q/D^{5/2}$ = 9.04). Similar discharges were set on the model. Fig. 28 shows comparative flows in the prototype and the model. Measurements were made of the locations and heights of the wave elements.

The measured distances to (x) and heights of (d_N) the prototype and model wave elements are given in Table 1 in terms of the pipe diameter (D). The differences are also listed.

Accurate measurements are very difficult to make. Flat crests and troughs and the judgment required to locate the nodes (N even) lend uncertainty to the measurement of the distances. Fluctuating wave depths covered with spray lend a similar uncertainty to the depth measurements. Fig. 28 gives only a rough idea of the difficulties of making accurate measurements. In view of these comments, the



(a) 3-ft. Diameter Prototype

(b) 4-in. Diameter Model

Fig. 28 - Comparison of the Waves in the Prototype and Its Model

TABLE 1

PROTOTYPE-MODEL COMPARISONS

N	x/D Prototype	x/D Model	x/D Difference	d_N/D Prototype	d_N/D Model	d_N/D Difference
			$Q/D^{5/2} = 9.04$			
1	4.27	3.90	-0.37	1.10	1.14	+0.04
2	7.73	7.11	-0.62	0.23	0.14	-0.09
3	9.93	10.26	+0.33	0.77	0.68	-0.09
4	12.67	13.50	+0.83	0.40	0.32	-0.08
5	14.73	15.00	+0.27	0.78	0.62	-0.16
			$Q/D^{5/2} = 12.6$			
1	5.60	6.15	+0.55	1.54	1.26	-0.28
2	11.10	10.95	-0.15	0.38	0.31	-0.07
3	14.30	14.16	-0.14	0.88	1.17	+0.29

agreement between the prototype and the model given in Table 1 is as good as can be expected. It is concluded, therefore, that the model predicts the locations and depths of the wave elements in the transition channel.

(a) Prototype

(b) Model

Fig. 29 - Comparison of the Expansion Section Performance in the Prototype and Its Model

A visual comparison of the spread of the jet as it passes through the expansion section is presented in Fig. 29. The angle iron posts shown in the prototype represent the beginning of the transition channel shown in the model. The posts and channel walls are 1.0 D apart and are located 0.5 D downstream from the pipe exit. As near as can be estimated from the width of the jet and the splash from the posts or channel entrance, the prototype and model give similar results. The splash shown is small, is contained within the expansion section, and will cause no problems in the prototype. These findings assure that the prototype expansion sections will perform as predicted by the models.

CONCLUSIONS AND RECOMMENDATIONS

The conclusions and recommendations resulting from the study of an abrupt transition from a circular pipe to a rectangular open channel are summarized in Fig. 30 and the following statements.

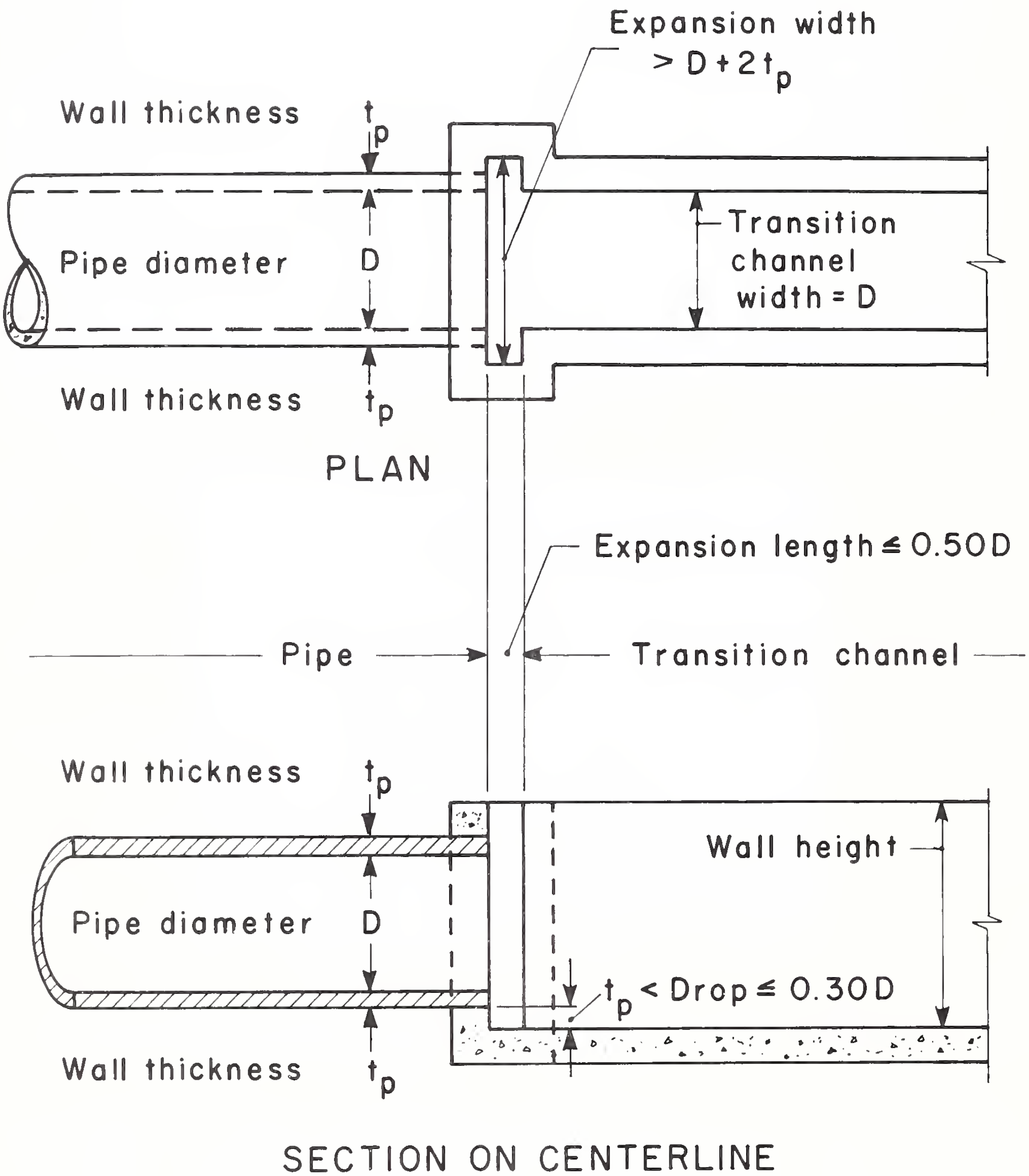


Fig. 30 - Recommended Dimensions

1. The distance between the transition channel sidewalls must be 1.0 D. Wider channels allow the water to spread and climb the sidewalls. The 1.0 D wide jet suppresses the sidewall waves when the channel is 1.0 D wide.

2. An expansion section may be used at the pipe exit to permit the pipe to expand longitudinally.
   a. The minimum length of the expansion section is determined by the anticipated pipe expansion.
   b. The length of the expansion section should not exceed 0.5 D in order to avoid excessive disturbances in the transition channel.
   c. The width of the expansion section should exceed the outside diameter of the pipe. The width is not controlled by hydraulic considerations.

3. To provide for pipe expansion, the transition floor should be below the bottom of the pipe.

4. The distance from the pipe exit to the elements comprising the water surface configuration are given by Eqs. 6 and 7. The equations will locate the surface elements within a maximum range of ± 1.4 D of their correct locations, except possibly for wall wave 1 when $y/D = 0.00$. The expected average error in locating the surface elements by Eqs. 6 and 7 will be about 0.11 D.

5. The depth of flow
   a. at the first node is given by Eqs. 11;
   b. at succeeding nodes and the average depth of flow are given by Eqs. 15, 11, 13, 6 and 7;
   c. will be given by these equations within $+0.13$ D and -0.06 D of the correct value. The average error will be about $+0.01$ D.

6. The maximum wave height at the sidewall and the minimum sidewall height
   a. for the first wall wave is given by Eq. 18;
   b. for the second wall wave is given by Eq. 19;
   c. for the third, fourth, and fifth wall waves is given by Eq. 20;
   d. given by Eqs. 18, 19 and 20 will provide a minimum freeboard of about zero, a maximum freeboard of about 0.3 D, and an average freeboard of about 0.08 D.

7. The centerline wave height, if required, can be computed using Eq. 15 and Fig. 27.

ACKNOWLEDGEMENTS

The study of the rectangular cantilevered outlet for pipe drop inlet spillways was first proposed for research in March 1962 by M. M. Culp, Chief, Design Branch, and was approved as a first-priority engineering research need on June 1, 1965 by C. J. Francis, Director, Engineering Division, Soil Conservation Service, U. S. Department of Agriculture.

The research was performed by Agricultural Research Service personnel at the St. Anthony Falls Hydraulic Laboratory in cooperation with the Minnesota Agricultural Experiment Station and the St. Anthony Falls Hydraulic Laboratory, University of Minnesota. The study was directed by Fred W. Blaisdell who planned the experiments, did part of the analysis, and wrote the paper. Charles A. Donnelly assembled the equipment, made the tests, and computed and plotted the data. Kesavarao Yalamanchili completed the analysis of the data, wrote the computer programs, and supervised much of the computation. All three engineers contributed to the ideas presented herein when discussing the analysis of the test results.

APPENDIX

SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS

	x/D			d/D			
N	TEST	EQ.	DIFF.	TEST	EQ.	DIFF.	DIFF.
		y/D = .00		$Q/D^{5/2}$ = 5.57			
1	.812	2.089	1.277	.969	1.023	.054	
2	2.120	2.785	.665	.774	.773		-.001
3	2.500	3.481	.981	.831			
4	3.500	4.177	.677	.774	.790		.016
5	4.370	4.874	.504	.831	.885	.054	
6	5.310	5.570	.260	.807	.816		.009
7	6.000	6.266	.266	.852			
8	6.440	6.962	.522	.819	.835		.016
9	7.250	7.659	.409	.879	.907	.028	
10	7.940	8.355	.415	.834	.867		.033
11	8.750	9.051	.301	.888			
12	9.250	9.747	.497	.852	.909		.057
13	10.060	10.444	.384	.936	1.025	.089	
		y/D = .00		$Q/D^{5/2}$ = 6.82			
1	1.060	2.557	1.497	1.032	1.073	.041	
2	2.620	3.410	.790	.813	.802		-.011
3	3.250	4.262	1.012	.873			
4	4.370	5.115	.745	.813	.816		.003
5	5.500	5.967	.467	.891	.927	.036	
6	6.500	6.820	.320	.837	.834		-.003
7	7.370	7.672	.302	.879			
8	8.000	8.525	.525	.849	.847		-.002
9	8.940	9.377	.438	.912	.925	.013	
10	9.750	10.230	.480	.864	.865		.001
11	10.750	11.082	.333	.906			
12	11.370	11.935	.565	.876	.884		.008
13	12.370	12.787	.417	.957	.965	.008	
14	13.370	13.640	.270	.894	.910		.016
15	14.250	14.492	.242	.918			
16	14.810	15.345	.535	.900	.933		.033
		y/D = .00		$Q/D^{5/2}$ = 8.29			
1	1.500	3.109	1.609	1.041	1.132	.091	
2	3.440	4.145	.705	.837	.831		-.006
3	4.690	5.181	.491	.888			
4	5.500	6.217	.718	.846	.842		-.004
5	7.000	7.254	.254	.912	.976	.064	
6	8.250	8.290	.040	.864	.859		-.005
7	9.250	9.326	.076	.900			
8	10.120	10.363	.243	.876	.871		-.005
9	11.000	11.399	.399	.924	.960	.036	
10	12.370	12.435	.065	.879	.887		.008
11	13.250	13.471	.221	.918			
12	14.250	14.508	.258	.894	.900		.006
13	15.620	15.544	-.076	.966	.994	.028	
		y/D = .00		$Q/D^{5/2}$ = 8.99			
1	1.690	3.371	1.681	1.050	1.160	.110	
2	3.560	4.495	.935	.849	.843		-.006
3	4.440	5.619	1.179	.912			
4	5.810	6.742	.932	.849	.854		.005
5	7.500	7.866	.366	.927	.998	.071	
6	8.560	8.990	.430	.876	.869		-.007
7	9.500	10.114	.614	.909			
8	10.250	11.237	.987	.885	.879		-.006
9	11.500	12.361	.861	.945	.976	.031	

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

N	x/D TEST	x/D EQ.	x/D DIFF.	d/D TEST	d/D EQ.	d/D DIFF.	DIFF.
		y/D = .00		$Q/D^{5/2}$ = 8.99			
10	12,560	13.485	.925	,891	,893		,002
11	13,500	14.609	1.109	,930			
12	14,750	15.732	.982	,900	,907		,007
13	15,750	16.856	1.106	,960	1,003	,043	
		y/D = .00		$Q/D^{5/2}$ = 9.41			
1	1,750	3.529	1.779	1,044	1,176	,132	
2	4,060	4.705	.645	,855	,850		-,005
3	4,940	5.881	.941	,921			
4	6,310	7.057	.747	,855	,860		,005
5	8,370	8.234	-.136	,924	1,012	,088	
6	9,190	9.410	.220	,870	,875		,005
7	9,870	10.586	.716	,912			
8	11,120	11.762	.642	,879	,885		,006
9	13,060	12.939	-.121	,948	,990	,042	
10	14,070	14.115	.045	,894	,902		,008
11	15,000	15.291	.291	,921			
12	16,060	16.467	.407	,897	,914		,017
13	17,060	17.644	.584	,960	1,014	,054	
		y/D = .00		$Q/D^{5/2}$ = 9.82			
1	2,000	3.682	1.682	1,044	1,193	,149	
2	4,250	4.910	.660	,858	,856		-,002
3	5,250	6.137	.887	,951			
4	6,750	7.365	.615	,861	,868		,007
5	8,500	8.592	.093	94,500	1,023	-93,477	
6	9,690	9.820	.130	,876	,882		,006
7	10,500	11.047	.547	,924			
8	11,500	12.275	.775	,888	,891		,003
9	13,500	13.502	.002	,951	1,000	,049	
10	14,500	14.730	.230	,897	,907		,010
11	15,500	15.957	.458	,930			
12	16,560	17.185	.625	,903	,919		,016
		y/D = .00		$Q/D^{5/2}$ = 10.15			
1	2,000	3.806	1.806	1,098	1,206	,108	
2	,100	5.075	4.975	,864	,861		-,003
3	5,370	6.344	.974	,957			
4	0	7.612	7.612	,864	,861		-,003
5	8,750	8.881	.131	,957	1,054	,097	
6	10,000	10.150	.150	,885	,908		,023
7	10,940	11.419	.479	,930			
8	12,000	12.687	.688	,894	,919		,025
9	13,750	13.956	.206	,963	1,030	,067	
10	14,750	15.225	.475	,900	,935		,035
11	16,000	16.494	.494	,933			
12	17,000	17.762	.762	,903	,948		,045
		y/D = .00		$Q/D^{5/2}$ = 10.53			
1	2,060	3.949	1.889	1,110	1,221	,111	
2	4,310	5.265	.955	,861	,867		,006
3	5,500	6.581	1.081	,939			
4	7,000	7.897	.897	,861	,878		,017
5	9,120	9.214	.094	,954	1,046	,092	
6	10,500	10.530	.030	,900	,894		-,006
7	11,750	11.846	.096	,924			
8	12,750	13.162	.413	,900	,905		,005

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

N	x/D TEST	x/D EQ.	x/D DIFF.	d/D TEST	d/D EQ.	d/D DIFF.	d/D DIFF.
		y/D = .00		$Q/D^{5/2}$ = 10.53			
9	14.500	14.479	-.021	.954	1.019	.065	
10	15.750	15.795	.045	.900	.920		.020
		y/D = .00		$Q/D^{5/2}$ = 11.80			
1	2.250	4.425	2.175	1.155	1.272	.117	
2	4.620	5.900	1.280	.876	.885		.009
3	6.000	7.375	1.375	.978			
4	7.620	8.850	1.230	.876	.896		.020
5	9.380	10.325	.945	.963	1.081	.118	
6	10.870	11.800	.930	.894	.910		.016
7	12.000	13.275	1.275	.945			
8	13.190	14.750	1.560	.903	.920		.017
9	14.940	16.225	1.285	.954	1.045	.091	
10	16.560	17.700	1.140	.909	.935		.026
		y/D = .00		$Q/D^{5/2}$ = 11.95			
1	2.370	4.481	2.111	1.140	1.278	.138	
2	5.000	5.975	.975	.879	.887		.008
3	6.250	7.469	1.219	.993			
4	7.870	8.962	1.092	.872	.898		.026
5	10.000	10.456	.456	.969	1.086	.117	
6	11.000	11.950	.950	.903	.910		.007
7	12.870	13.444	.574	.933			
8	14.120	14.937	.817	.906	.924		.018
9	15.870	16.431	.561	.963	1.051	.088	
		y/D = .00		$Q/D^{5/2}$ = 12.40			
1	2.620	4.650	2.030	1.143	1.296	.153	
2	4.940	6.200	1.260	.882	.893		.011
3	6.560	7.750	1.190	1.029			
4	8.690	9.300	.610	.891	.907		.016
5	10.750	10.850	.100	.972	1.101	.129	
6	12.000	12.400	.400	.894	.920		.026
7	13.120	13.950	.830	.963			
8	14.810	15.500	.690	.891	.932		.041
9	16.690	17.050	.360	.963	1.064	.101	
		y/D = .00		$Q/D^{5/2}$ = 14.00			
1	3.250	5.250	2.000	1.206	1.360	.154	
2	6.000	7.000	1.000	.894	.913		.019
3	7.500	8.750	1.250	1.050			
4	9.620	10.500	.880	.894	.925		.031
5	12.250	12.250	0	.999	1.145	.146	
6	13.750	14.000	.250	.900	.940		.040
7	15.250	15.750	.500	.954			
8	16.500	17.500	1.000	.900	.951		.051
		y/D = .00		$Q/D^{5/2}$ = 15.52			
1	3.620	5.820	2.200	1.269	1.421	.152	
2	6.500	7.760	1.260	.900	.930		.030
3	8.250	9.700	1.450	1.080			
4	11.250	11.640	.390	.918	.945		.027
5	13.000	13.580	.580	.984	1.184	.200	
6	14.750	15.520	.770	.918	.958		.040
7	16.120	17.460	1.340	.960			

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

	x/D			d/D			
N	TEST	EQ.	DIFF.	TEST	EQ.	DIFF.	DIFF.
		y/D = .00		$Q/D^{5/2}$ = 17.38			
1	4,500	6,518	2,018	1,302	1,495	,193	
2	8,060	8,690	.630	,918	,949		,031
3	9,500	10,862	1.362	1,083			
4	12,000	13,035	1.035	,918	,961		,043
5	14,310	15,207	.897	,993	1,230	,237	
6	16,750	17,380	.630	,918	,977		,059
		y/D = .00		$Q/D^{5/2}$ = 18.34			
1	4,810	6,877	2,067	1,326	1,534	,208	
2	8,190	9,170	.980	,927	,958		,031
3	10,120	11,462	1,342	1,104			
4	13,500	13,755	.255	,927	,975		,048
5	16,750	16,047	-.703	1,008	1,261	,253	
		y/D = .00		$Q/D^{5/2}$ = 18.90			
1	5,120	7,087	1.967	1,356	1,556	,200	
2	8,430	9.450	1.020	,933	,963		,030
3	10,500	11,812	1.312	1,149			
4	13,750	14,175	.425	,936	,980		,044
5	17,000	16,538	-.462	1,020	1,274	,254	
		y/D = .05		$Q/D^{5/2}$ = 4.34			
1	,562	,328	-.234	1,029	1,050	,021	
2	2,000	1.140	-.860	,747	,716		-,031
3	2,310	1.951	-.359	,789			
4	3,380	2,763	-.617	,723	,748		,025
5	4,430	3,460	-.970	,804	,854	,050	
		y/D = .05		$Q/D^{5/2}$ = 5.36			
1	,687	,709	.022	1,053	1,050	-,003	
2	2,250	1.646	-.604	,780	,751		-,029
3	2,560	2.584	.024	,813			
4	3,560	3,522	-.038	,753	,766		,013
5	4,690	4,327	-.363	,837	,861	,024	
6	5,310	5,132	-.178	,792	,789		-,003
7	5,750	5,936	.186	,828			
8	6,560	6,741	.181	,804	,809		,005
9	7,500	7,546	.046	,870	,881	,011	
10	8,060	8,351	.291	,831	,840		,009
11	8,560	9,156	.596	,867			
12	9,250	9,961	.711	,840	,874		,034
13	10,190	10,766	.576	,924	,975	,051	
		y/D = .05		$Q/D^{5/2}$ = 6.82			
1	,880	1,254	.374	1,065	1,073	,008	
2	2,690	2,372	-.318	,825	,792		-,033
3	3,250	3,490	.240	,867			
4	4,500	4,608	.108	,810	,806		-,004
5	5,750	5,567	-.183	,900	,918	,018	
6	6,560	6,527	-.033	,843	,822		-,021
7	7,250	7,486	.236	,882			
8	8,000	8,446	.446	,843	,834		-,009
9	9,250	9,406	.156	,915	,914	-,001	
10	9,750	10,365	.615	,864	,850		-,014

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

	x/D			d/D			
N	TEST	EQ.	DIFF.	TEST	EQ.	DIFF.	DIFF.
		y/D = .05		$Q/D^{5/2}$ = 6.82			
11	10,500	11.325	.825	,906			
12	11,250	12.285	1.035	,888	,866		-,022
13	12,440	13.244	.804	,945	,948	,003	
14	13,250	14.204	.954	,894	,889		-,005
15	14,120	15.164	1.044	,930			
16	14,940	16.123	1.183	,885	,912		,027
		y/D = .05		$Q/D^{5/2}$ = 8.29			
1	1,250	1.803	.553	1,062	1,132	,070	
2	3,310	3.102	-.208	,852	,828		-,024
3	4,000	4.402	.402	,915			
4	5,250	5.701	.451	,840	,839		-,001
5	6,750	6.816	.066	,942	,972	,030	
6	8,000	7.932	-.068	,876	,855		-,021
7	8,500	9.047	.547	,921			
8	9,560	10.163	.603	,885	,865		-,020
9	10,250	11.278	1.028	,942	,952	,010	
10	11,750	12.393	.643	,900	,879		-,021
11	12,250	13.509	1.259	,945			
12	13,620	14.624	1.004	,900	,892		-,008
13	14,940	15.740	.800	,969	,985	,016	
		y/D = .05		$Q/D^{5/2}$ = 8.99			
1	1,750	2.064	.314	1,065	1,160	,095	
2	3,750	3.450	-.300	,867	,843		-,024
3	4,750	4.836	.086	,946			
4	6,250	6.221	-.029	,855	,856		,001
5	7,750	7.411	-.339	,945	,998	,053	
6	9,000	8.601	-.399	,888	,870		-,018
7	9,750	9.790	.040	,942			
8	10,880	10.980	.100	,894	,881		-,013
9	12,000	12.170	.170	,957	,978	,021	
10	13,380	13.359	-.021	,915	,896		-,019
11	14,250	14.549	.299	,951			
		y/D = .05		$Q/D^{5/2}$ = 9.41			
1	2,000	2.221	.221	1,104	1,176	,072	
2	4,250	3.659	-.591	,873	,850		-,023
3	5,120	5.096	-.024	,975			
4	6,440	6.534	.094	,870	,860		-,010
5	8,000	7.768	-.232	,987	1,009	,022	
6	9,370	9.002	-.368	,900	,875		-,025
7	10,250	10.236	-.014	,945			
8	11,440	11.470	.030	,900	,886		-,014
9	12,750	12.704	-.046	,972	,987	,015	
10	13,880	13.939	.059	,909	,900		-,009
11	15,370	15.173	-.197	,919			
		y/D = .05		$Q/D^{5/2}$ = 9.82			
1	2,250	2.374	.124	1,140	1,193	,053	
2	4,180	3.862	-.318	,875	,856		-,019
3	5,370	5.351	-.019	1,020			
4	6,750	6.839	.089	,879	,868		-,011
5	8,560	8.116	-.444	1,005	1,024	,019	
6	9,880	9.394	-.486	,909	,883		-,026
7	10,750	10.671	-.079	,951			

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

	x/D			d/D			
N	TEST	EQ.	DIFF.	TEST	EQ.	DIFF.	DIFF.
		$y/D = .05$		$Q/D^{5/2} = 9.82$			
8	12,060	11.949	-.111	.912	.894		-.018
9	13,500	13.227	-.273	.990	1.000	.010	
10	14,500	14.504	.004	.915	.907		-.008
		$y/D = .05$		$Q/D^{5/2} = 10.15$			
1	2,250	2.498	.248	1.173	1.206	.033	
2	4,250	4.026	-.224	.888	.861		-.027
3	5,370	5.555	.185	1.029			
4	7,120	7.084	-.036	.888	.874		-.014
5	8,620	8.397	-.223	1.008	1.033	.025	
6	10,000	9.709	-.291	.915	.887		-.028
7	11,180	11.022	-.158	.966			
8	12,180	12.334	.154	.915	.898		-.017
9	13,880	13.647	-.233	1.005	1.008	.003	
10	14,880	14.959	.079	.905	.912		.007
		$y/D = .05$		$Q/D^{5/2} = 10.53$			
1	2,500	2.639	.139	1.194	1.221	.027	
2	4,430	4.215	-.215	.888	.867		-.021
3	5,500	5.791	.291	1.035			
4	7,000	7.367	.367	.894	.878		-.016
5	9,000	8.720	-.280	.972	1.045	.073	
6	10,250	10.072	-.178	.936	.892		-.044
7	11,500	11.425	-.075	.966			
8	12,370	12.778	.408	.921	.902		-.019
9	14,000	14.131	.131	.984	1.016	.032	
10	15,250	15.484	.234	.930	.917		-.013
		$y/D = .05$		$Q/D^{5/2} = 11.03$			
1	2,750	2.826	.076	1.200	1.241	.041	
2	4,500	4.464	-.036	.894	.874		-.020
3	6,000	6.101	.101	1.050			
4	7,750	7.739	-.011	.906	.888		-.018
5	9,500	9.145	-.355	.981	1.060	.079	
6	10,750	10.550	-.200	.942	.900		-.042
7	11,940	11.956	.016	.969			
8	13,250	13.362	.112	.921	.912		-.009
9	14,750	14.768	.018	.990	1.029	.039	
		$y/D = .05$		$Q/D^{5/2} = 12.40$			
1	3,000	3.338	.338	1.242	1.296	.054	
2	5,120	5.144	.024	.894	.893		-.001
3	6,750	6.951	.201	1.083			
4	8,620	8.758	.138	.885	.906		.021
5	10,620	10.309	-.311	.990	1.100	.110	
6	12,000	11.860	-.140	.936	.919		-.017
7	12,750	13.411	.661	.969			
8	14,500	14.961	.461	.915	.929		.014
		$y/D = .05$		$Q/D^{5/2} = 14.00$			
1	3,500	3.935	.435	1.272	1.360	.088	
2	5,750	5.939	.189	.885	.913		.028
3	7,750	7.944	.194	1.098			
4	9,620	9.948	.328	.897	.926		.029
5	11,500	11.668	.168	1.005	1.143	.138	

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

N	x/D TEST	x/D EQ.	x/D DIFF.	d/D TEST	d/D EQ.	d/D DIFF.	d/D DIFF.
		y/D = .05		$Q/D^{5/2}$ = 14.00			
6	13,370	13.389	.019	,942	,940		-,002
7	14,620	15.109	.489	,969			
		y/D = .05		$Q/D^{5/2}$ = 15.52			
1	3,870	4.503	.633	1,314	1,421	,107	
2	6,250	6.695	.445	,897	,930		,033
3	8,750	8.886	.136	1,140			
4	10,500	11.078	.578	,912	,944		,032
5	12,500	12.960	.460	1,020	1,183	,163	
6	14,190	14.841	.651	,930	,957		,027
7	15,750	16.723	.973	,975			
		y/D = .05		$Q/D^{5/2}$ = 17.02			
1	4,500	5.063	.563	1,350	1,481	,131	
2	7,620	7.440	-.180	,930	,945		,015
3	10,250	9.817	-.433	1,209			
4	12,500	12.194	-.306	,936	,961		,025
5	14.430	14.234	-.196	1,041	1,223	,182	
		y/D = .05		$Q/D^{5/2}$ = 18.87			
1	5,250	5.754	.504	1,425	1,555	,130	
2	8,680	8.359	-.321	,945	,963		,018
3	11,750	10.964	-.786	1,260			
4	14,500	13.570	-.930	,945	,981		,036
		y/D = .10		$Q/D^{5/2}$ = 4.34			
1	,500	,259	-.241	1,044	1,100	,056	
2	1,940	1.174	-.766	,717	,694		-,023
3	2,370	2.088	-.282	,705			
4	3,250	3.002	-.248	,657	,716		,059
5	4,190	3.731	-.459	,720	,801	,081	
6	4,870	4.460	-.410	,684	,754		,070
7	5,500	5.189	-.311	,726			
8	5,940	5.918	-.022	,705	,805		,100
		y/D = .10		$Q/D^{5/2}$ = 5.36			
1	,625	,639	.014	1,086	1,100	,014	
2	2,370	1.677	-.693	,765	,734		-,031
3	2,810	2.716	-.094	,789			
4	4,000	3.755	-.245	,735	,750		,015
5	4,870	4.583	-.287	,807	,841	,034	
6	5,620	5.411	-.209	,765	,769		,004
7	6,000	6.239	.239	,801			
8	7,000	7.067	.067	,780	,788		,008
9	8,000	7.895	-.105	,843	,857	,014	
10	8,690	8.723	.033	,798	,816		,018
11	9,250	9.551	.301	,834			
12	10,060	10.380	.320	,816	,845		,029
13	10,870	11.208	.338	,876	,922	,046	
14	11,620	12.036	.416	,840	,898		,058
15	12,190	12.864	.674	,876			

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

N	x/D TEST	x/D EQ.	x/D DIFF.	d/D TEST	d/D EQ.	d/D DIFF.	d/D DIFF.
		y/D = .10		$Q/D^{5/2}$ = 6.82			
1	,870	1.182	.312	1,080	1,100	,020	
2	2,810	2.398	-.412	,819	,783		-,036
3	3,500	3.615	.115	,849			
4	4,620	4.831	.211	,798	,795		-,003
5	5,940	5.801	-.139	,873	,907	,034	
6	6,940	6.771	-.169	,831	,812		-,019
7	7,500	7.741	.241	,870			
8	8,440	8.711	.271	,840	,825		-,015
9	9,500	9.681	.181	,900	,902	,002	
10	10,500	10.651	.151	,861	,843		-,018
11	11,120	11.621	.501	,900			
12	12,000	12.591	.591	,870	,858		-,012
13	13,000	13.561	.561	,936	,936	,000	
14	13,940	14.531	.591	,876	,879		,003
15	14,750	15.501	.751	,900			
		y/D = .10		$Q/D^{5/2}$ = 8.29			
1	1,440	1.728	.288	1,035	1,132	,097	
2	3,620	3.124	-.496	,846	,825		-,021
3	4.500	4.520	.020	,888			
4	5,750	5.916	.166	,834	,836		,002
5	7,250	7.028	-.222	,936	,969	,033	
6	8,000	8.141	.141	,855	,849		-,006
7	8,440	9.254	.814	,900			
8	9,810	10.367	.557	,864	,861		-,003
9	11,000	11.480	.480	,933	,951	,018	
10	11,870	12.592	.722	,888	,874		-,014
11	12,750	13.705	.955	,921			
12	13,620	14.818	1.198	,888	,886		-,002
13	14,500	15.931	1.431	,954	,975	,021	
		y/D = .10		$Q/D^{5/2}$ = 8.99			
1	1,810	1.989	.179	1,023	1,160	,137	
2	3,870	3.470	-.400	,852	,843		-,009
3	4,870	4.951	.081	,933			
4	6,620	6.432	-.188	,846	,857		,011
5	7,810	7.613	-.197	,954	,998	,044	
6	9,000	8.793	-.207	,876	,870		-,006
7	9,940	9.974	.034	,915			
8	10,940	11.155	.215	,876	,881		,005
9	12,000	12.336	.336	,957	,977	,020	
10	13,250	13.517	.267	,900	,895		-,005
11	14,000	14.698	.698	,942			
12	15,250	15.878	.628	,900	,908		,008
		y/D = .10		$Q/D^{5/2}$ = 9.41			
1	2,060	2.145	.085	1,032	1,176	,144	
2	4,190	3.677	-.513	,852	,850		-,002
3	5,000	5.209	.209	,954			
4	6,940	6.742	-.198	,842	,863		,021
5	8,370	7.963	-.407	,966	1,011	,045	
6	9,500	9.185	-.315	,876	,876		-,000
7	10,440	10.406	-.034	,936			
8	11,500	11.628	.128	,885	,887		,002

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

N	x/D TEST	x/D EQ.	x/D DIFF.	d/D TEST	d/D EQ.	d/D DIFF.	d/D DIFF.
		y/D = .10		$Q/D^{5/2}$ = 9.41			
9	13,000	12,850	-.150	,960	,989	,029	
10	14,120	14,071	-,049	,903	,901		-,002
11	15,250	15.293	.043	,939			
		y/D = .10		$Q/D^{5/2}$ = 9.82			
1	2,190	2,297	.107	1,041	1,193	,152	
2	4,310	3,879	-.431	,870	,856		-,014
3	5,310	5.462	.152	,963			
4	6,940	7,044	.104	,861	,868		,007
5	8,500	8,305	-.195	,966	1,023	,057	
6	9,810	9.567	-.243	,894	,882		-,012
7	10,750	10,828	.078	,951			
8	12,000	12,090	.090	,888	,893		,005
9	13,250	13,351	.101	,963	,998	,035	
10	14,500	14,613	.113	,903	,907		,004
11	15,250	15.874	.624	,936			
		y/D = .10		$Q/D^{5/2}$ = 10.53			
1	2,310	2.561	.251	1,095	1,221	,126	
2	4,500	4,230	-.270	,873	,867		-,006
3	5,560	5,899	.339	,972			
4	7,250	7,568	.318	,858	,879		,021
5	9,000	8,898	-.102	,981	1,044	,063	
6	10,310	10,228	-.082	,903	,892		-,011
7	11,500	11,559	.059	,930			
8	12,370	12,889	.519	,894	,902		,008
9	13,690	14,220	.530	,960	1,014	,054	
		y/D = .10		$Q/D^{5/2}$ = 12.40			
1	3,000	3,257	.257	1,164	1,296	,132	
2	5,500	5,153	-.347	,861	,893		,032
3	6,870	7,050	.180	1,044			
4	8,810	8,947	.137	,861	,905		,044
5	11,000	10,459	-.541	,990	1,100	,110	
6	12,750	11,971	-.779	,915	,921		,006
7	13,620	13,483	-.137	,927			
		y/D = .10		$Q/D^{5/2}$ = 14.00			
1	3,750	3,852	.102	1,272	1,360	,088	
2	6,370	5,943	-.427	,876	,913		,037
3	8,120	8,035	-.085	1,110			
4	10,440	10,127	-.313	,876	,927		,051
5	12,500	11,794	-.706	1,008	1,144	,136	
6	14,190	13,462	-.728	,912	,940		,028
7	15,620	15,130	-.490	,948			
		y/D = .10		$Q/D^{5/2}$ = 15.52			
1	4,250	4,417	.167	1,350	1,421	,071	
2	7,120	6,694	-.426	,900	,930		,030
3	9,000	8.971	-.029	1,170			
4	11,250	11.248	-.002	,900	,943		,043
5	13,250	13.063	-.187	1,032	1,183	,151	

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

N	x/D TEST	x/D EQ.	x/D DIFF.	d/D TEST	d/D EQ.	d/D DIFF.	d/D DIFF.
		y/D = .10		$Q/D^{5/2}$ = 17.02			
1	4,750	4.975	.225	1,410	1,481	,071	
2	7,940	7.434	-.506	,921	,945		.024
3	10,250	9.894	-.356	1,200			
4	12,000	12.354	.354	,927	,958		.031
5	14,940	14.315	-.625	1,044	1,223	,179	
		y/D = .10		$Q/D^{5/2}$ = 18.87			
1	5,500	5.663	.163	1,410	1,555	,145	
2	9,000	8.348	-.652	,945	,963		.018
3	11,750	11.033	-.717	1,260			
4	14,000	13.718	-.282	,945	,979		.034
		y/D = .15		$Q/D^{5/2}$ = 4.34			
1	,440	.241	-.199	1,092	1,150	,058	
2	1,870	1.229	-.641	,636	,672		.036
3	2,190	2.217	.027	,681			
4	3,000	3.205	.205	,615	,686		.071
5	4,250	3.945	-.305	,675	,771	,096	
6	4,750	4.684	-.066	,642	,714		.072
7	5,500	5.424	-.076	,675			
8	6,000	6.164	.164	,654	,742		.088
9	6,750	6.903	.153	,705	,808	,103	
10	7,500	7.643	.143	,672	,805		.133
		y/D = .15		$Q/D^{5/2}$ = 5.36			
1	,560	.619	.059	1,146	1,150	,004	
2	2,250	1.729	-.521	,765	,717		-.048
3	2,560	2.840	.280	,786			
4	3,940	3.951	.011	,714	,732		.018
5	5,000	4.783	-.217	,795	,823	,028	
6	5,750	5.614	-.136	,765	,751		-.014
7	6,370	6.446	.076	,786			
8	6,810	7.277	.467	,774	,763		-.011
9	7,870	8.108	.238	,822	,830	,008	
10	8,560	8.940	.380	,777	,785		.008
11	9,310	9.771	.461	,825			
12	10,120	10.603	.483	,810	,810		.000
13	10,870	11.434	.564	,876	,878	,002	
14	11,870	12.266	.396	,831	,847		.016
15	12,500	13.097	.597	,849			
16	13,000	13.929	.929	,834	,883		.049
17	13,750	14.760	1.010	,885	,980	,095	
		y/D = .15		$Q/D^{5/2}$ = 6.82			
1	,810	1.159	.349	1,149	1,150	,001	
2	2,810	2.446	-.364	,807	,773		-.034
3	3,250	3.732	.482	,837			
4	4,620	5.019	.399	,780	,785		.005
5	6,000	5.982	-.018	,870	,897	,027	
6	6,810	6.945	.135	,838	,800		-.038
7	7,310	7.908	.598	,849			
8	8,190	8.871	.681	,822	,811		-.011
9	9,370	9.834	.464	,885	,888	,003	
10	10,190	10.796	.606	,846	,827		-.019
11	10,870	11.759	.889	,879			
12	11,500	12.722	1.222	,855	,838		-.017
13	12,750	13.685	.935	,924	,918	-,006	

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

	x/D			d/D			
N	TEST	EQ.	DIFF.	TEST	EQ.	DIFF.	DIFF.
		$y/D = .15$		$Q/D^{5/2} = 6.82$			
14	13,560	14.648	1.088	,858	,858		,000
15	14,250	15.611	1.361	,885			
16	15,250	16.574	1.324	,858	,877		,019
		$y/D = .15$		$Q/D^{5/2} = 8.29$			
1	1,190	1.704	.514	1,098	1,150	,052	
2	3,250	3.167	-.083	,840	,822		-,018
3	4,250	4.630	.380	,891			
4	5,560	6.094	.534	,819	,834		,015
5	6,750	7.189	.439	,921	,965	,044	
6	7,870	8.284	.414	,855	,847		-,008
7	8,750	9.380	.630	,885			
8	9,560	10.475	.915	,852	,858		,006
9	10,500	11.570	1.070	,924	,947	,023	
10	11,500	12.666	1.166	,876	,870		-,006
11	12,810	13.761	.951	,906			
12	13,560	14.856	1.296	,876	,884		,008
13	14,500	15.952	1.452	,936	,974	,038	
		$y/D = .15$		$Q/D^{5/2} = 8.99$			
1	1,560	1.963	.403	1,050	1,160	,110	
2	4,060	3.510	-.550	,864	,843		-,021
3	4,750	5.058	.308	,945			
4	6,310	6.606	.296	,849	,854		,005
5	7,750	7.764	.014	,975	,997	,022	
6	9,120	8.922	-.198	,885	,869		-,016
7	10,250	10.081	-.169	,930			
8	11,000	11.239	.239	,885	,880		-,005
9	12,500	12.397	-.103	,963	,979	,016	
10	13,500	13.556	.056	,903	,895		-,008
11	14,690	14.714	.024	,942			
		$y/D = .15$		$Q/D^{5/2} = 9.41$			
1	1,810	2.118	.308	1,038	1,176	,138	
2	4,120	3.716	-.404	,849	,850		,001
3	5,060	5.315	.255	,939			
4	6,620	6.913	.293	,840	,862		,022
5	8,120	8.109	-.011	,984	1,010	,026	
6	9,250	9.305	.055	,891	,875		-,016
7	10,370	10.501	.131	,930			
8	11,250	11.697	.447	,885	,886		,001
9	12,750	12.894	.144	,984	,988	,004	
10	13,750	14.090	.340	,909	,900		-,009
11	14,750	15.286	.536	,933			
		$y/D = .15$		$Q/D^{5/2} = 9.82$			
1	2,120	2.270	.150	1,020	1,193	,173	
2	4,120	3.918	-.202	,849	,856		,007
3	5,190	5.565	.375	,939			
4	6,620	7.212	.592	,849	,868		,019
5	8,250	8.446	.196	,975	1,023	,048	
6	9,370	9.679	.309	,876	,881		,005
7	10,500	10.912	.412	,939			
8	11,500	12.145	.645	,894	,892		-,002
9	12,940	13.378	.438	,966	,997	,031	
10	14,000	14.611	.611	,888	,905		,017
11	15,250	15.844	.594	,930			

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

N	x/D TEST	x/D EQ.	x/D DIFF.	d/D TEST	d/D EQ.	d/D DIFF.	d/D DIFF.
		y/D = .15		$Q/D^{5/2}$ = 10.15			
1	2,250	2,392	.142	1,047	1,206	,159	
2	4,250	4,080	-.170	,864	,861		-,003
3	5,370	5,767	.397	,984			
4	7,000	7,454	.454	,858	,873		,015
5	8,430	8,717	.287	,990	1,032	,042	
6	9,620	9,979	.359	,894	,886		-,008
7	10,750	11,242	.492	,945			
8	11,680	12,505	.825	,897	,896		-,001
9	13,250	13,768	.518	,972	1,005	,033	
10	14,370	15,031	.661	,906	,909		,003
11	15,500	16,294	.794	,936			
		y/D = .15		$Q/D^{5/2}$ = 10.53			
1	2,500	2,533	.033	1,056	1,221	,165	
2	4,370	4,266	-.104	,864	,867		,003
3	5,500	5,999	.499	,981			
4	7,250	7,732	.482	,870	,879		,009
5	8,750	9,029	.279	,996	1,044	,048	
6	9,870	10,326	.456	,906	,891		-,015
7	11,250	11,623	.373	,960			
8	12,190	12,920	.730	,900	,902		,002
9	13,500	14,217	.717	,987	1,014	,027	
10	14,750	15,514	.764	,915	,914		-,001
		y/D = .15		$Q/D^{5/2}$ = 11.03			
1	2,690	2,718	.028	1,065	1,241	,176	
2	4,560	4,511	-.049	,879	,874		-,005
3	5,750	6,304	.554	,993			
4	7,440	8,097	.657	,879	,886		,007
5	9,000	9,439	.439	1,005	1,058	,053	
6	10,250	10,781	.531	,912	,898		-,014
7	11,690	12,124	.434	,954			
8	12,690	13,466	.776	,909	,909		-,000
9	14,120	14,808	.688	,996	1,026	,030	
10	15,370	16,150	.780	,912	,922		,010
		y/D = .15		$Q/D^{5/2}$ = 12.40			
1	3,370	3,226	-.144	1,200	1,296	,096	
2	5,370	5,183	-.187	,897	,893		-,004
3	6,870	7,141	.271	1,110			
4	8,690	9,099	.409	,900	,905		,005
5	10,560	10,565	.005	1,011	1,098	,087	
6	12,000	12,030	.030	,930	,918		-,012
7	13,500	13,496	-.004	,975			
8	14,940	14,961	.021	,915	,930		,015
		y/D = .15		$Q/D^{5/2}$ = 14.00			
1	3,750	3,818	.068	1,230	1,360	,130	
2	5,870	5,968	.098	,903	,913		,010
3	7,750	8,119	.369	1,125			
4	9,750	10,269	.519	,906	,926		,020
5	11,500	11,879	.379	1,020	1,142	,122	
6	13,000	13,488	.488	,924	,938		,014
7	14,560	15,098	.538	,981			

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

N	x/D TEST	x/D EQ.	x/D DIFF.	d/D TEST	d/D EQ.	d/D DIFF.	d/D DIFF.
		y/D = .15		$Q/D^{5/2}$ = 15.52			
1	4.370	4.381	.011	1.320	1.421	.101	
2	6.810	6.714	-.096	.915	.930		.015
3	8.750	9.047	.297	1.161			
4	11.060	11.381	.321	.915	.944		.029
5	12.750	13.127	.377	1.080	1.182	.102	
6	14.370	14.873	.503	.927	.955		.028
		y/D = .15		$Q/D^{5/2}$ = 17.02			
1	4.870	4.936	.066	1.434	1.481	.047	
2	7.870	7.450	-.420	.945	.945		.000
3	10.250	9.964	-.286	1.284			
4	12.370	12.477	.107	.960	.960		-.000
5	14.250	14.359	.109	1.095	1.221	.126	
		y/D = .15		$Q/D^{5/2}$ = 18.87			
1	5.870	5.622	-.248	1.487	1.555	.068	
2	9.060	8.358	-.702	.960	.963		.003
3	11.500	11.094	-.406	1.350			
4	15.120	13.830	-1.290	.990	.982		-.008
		y/D = .20		$Q/D^{5/2}$ = 4.34			
1	.440	.240	-.200	1.131	1.200	.069	
2	2.120	1.287	-.833	.690	.650		-.040
3	2.620	2.333	-.287	.726			
4	3.000	3.379	.379	.666	.660		-.006
5	5.000	4.118	-.882	.735	.749	.014	
6	5.500	4.857	-.643	.705	.691		-.014
7	6.000	5.595	-.405	.729			
8	6.750	6.334	-.416	.702	.711		.009
9	7.500	7.072	-.428	.756	.769	.013	
10	8.310	7.811	-.499	.720	.745		.025
11	8.870	8.549	-.321	.762			
12	9.810	9.288	-.522	.741	.823		.082
		y/D = .20		$Q/D^{5/2}$ = 5.36			
1	.500	.616	.116	1.140	1.200	.060	
2	2.250	1.784	-.466	.723	.701		-.022
3	3.000	2.951	-.049	.750			
4	3.940	4.119	.179	.705	.714		.009
5	5.250	4.943	-.307	.762	.806	.044	
6	5.810	5.767	-.043	.741	.731		-.010
7	6.370	6.591	.221	.762			
8	7.000	7.415	.415	.744	.743		-.001
9	8.000	8.239	.239	.801	.807	.006	
10	8.690	9.063	.373	.756	.761		.005
11	9.440	9.887	.447	.789			
12	10.180	10.711	.531	.777	.780		.003
13	11.000	11.535	.535	.831	.845	.014	
14	11.940	12.359	.419	.804	.807		.003
15	12.440	13.183	.743	.825			
16	13.120	14.008	.888	.804	.829		.025
17	14.250	14.832	.582	.861	.910	.049	
		y/D = .20		$Q/D^{5/2}$ = 6.82			
1	.750	1.155	.405	1.191	1.200	.009	
2	2.810	2.496	-.314	.786	.764		-.022

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

	x/D			d/D			
N	TEST	EQ.	DIFF.	TEST	EQ.	DIFF.	DIFF.
		$y/D = .20$		$Q/D^{5/2} = 6.82$			
3	3,500	3.837	.337	,831			
4	4,560	5.177	.617	,768	,775		,007
5	6,000	6.124	.124	,855	,886	,031	
6	6,870	7.070	.200	,807	,790		-,017
7	7,620	8.017	.397	,840			
8	8,620	8.963	.343	,804	,802		-,002
9	9,810	9.910	.100	,879	,879	,000	
10	10,440	10.856	.416	,831	,816		-,015
11	11,120	11.802	.682	,876			
12	12,120	12.749	.629	,843	,830		-,013
13	13,310	13.695	.385	,924	,909	-,015	
14	14,000	14.642	.642	,855	,847		-,008
15	14,940	15.588	.648	,879			
		$y/D = .20$		$Q/D^{5/2} = 8.29$			
1	1,120	1.697	.577	1,101	1,200	,099	
2	3,690	3.212	-.478	,840	,819		-,021
3	4,440	4.728	.288	,903			
4	5,810	6.243	.433	,825	,830		,005
5	7,750	7.313	-.437	,927	,965	,038	
6	8,370	8.382	.012	,855	,844		-,011
7	9,750	9.452	-.298	,888			
8	10,500	10.522	.022	,867	,857		-,010
9	11,870	11.591	-.279	,992	,949	-,043	
10	12,500	12.661	.161	,873	,870		-,003
11	13,250	13.731	.481	,915			
12	14,440	14.800	.360	,882	,883		,001
13	15,620	15.870	.250	,951	,975	,024	
		$y/D = .20$		$Q/D^{5/2} = 8.99$			
1	1,440	1.955	.515	1,074	1,200	,126	
2	4,000	3.553	-.447	,852	,843		-,009
3	4,870	5.152	.282	,930			
4	6,120	6.751	.631	,855	,853		-,002
5	8,060	7.879	-.181	,954	,999	,045	
6	9,120	9.007	-.113	,870	,870		-,000
7	10,120	10.136	.016	,915			
8	10,810	11.264	.454	,879	,879		,000
9	12,620	12.392	-.228	,954	,980	,026	
10	13,310	13.520	.210	,894	,894		,000
11	14,310	14.649	.339	,924			
12	15,310	15.777	.467	,894	,907		,013
		$y/D = .20$		$Q/D^{5/2} = 9.41$			
1	1,750	2.110	.360	1,044	1,200	,156	
2	4,440	3.758	-.682	,864	,850		-,014
3	5,370	5.407	.037	,969			
4	6,810	7.055	.245	,864	,861		-,003
5	8,750	8.219	-.531	,972	1,012	,040	
6	9,810	9.382	-.428	,894	,876		-,018
7	11,000	10.546	-.454	,939			
8	11,810	11.709	-.101	,897	,887		-,010
9	13,250	12.873	-.377	,954	,989	,035	
10	14,500	14.036	-.464	,906	,902		-,004

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

N	x/D			d/D			
	TEST	EQ.	DIFF.	TEST	EQ.	DIFF.	DIFF.
		$y/D = .20$		$Q/D^{5/2} = 9.82$			
1	2,000	2.261	.261	1,026	1,200	,174	
2	4,500	3.958	-.542	,875	,856		-,019
3	5,500	5.655	.155	,960			
4	6,940	7.352	.412	,845	,867		,022
5	8,750	8.550	-.200	,990	1,023	,033	
6	10,000	9.748	-.252	,900	,882		-,018
7	11,120	10.946	-.174	,942			
8	12,000	12.144	.144	,900	,892		-,008
9	13,500	13.342	-.158	,966	,998	,032	
10	14,690	14.540	-.150	,906	,907		,001
		$y/D = .20$		$Q/D^{5/2} = 10.15$			
1	2,440	2.383	-.057	1,053	1,206	,153	
2	4,690	4.119	-.571	,876	,861		-,015
3	5,690	5.855	.165	,984			
4	7,310	7.592	.282	,879	,873		-,006
5	9,250	8.817	-.433	1,005	1,034	,029	
6	10,250	10.043	-.207	,915	,886		-,029
7	11,620	11.268	-.352	,957			
8	12,690	12.494	-.196	,915	,898		-,017
9	14,250	13.719	-.531	,981	1,008	,027	
10	15,250	14.945	-.305	,921	,912		-,009
		$y/D = .20$		$Q/D^{5/2} = 10.53$			
1	2,620	2.523	-.097	1,071	1,221	,150	
2	4,810	4.304	-.506	,894	,867		-,027
3	6,000	6.086	.086	1,026			
4	7,620	7.867	.247	,885	,879		-,006
5	9,500	9.125	-.375	1,011	1,045	,034	
6	10,500	10.382	-.118	,921	,892		-,029
7	11,750	11.639	-.111	,978			
8	13,000	12.897	-.103	,921	,904		-,017
9	14,750	14.154	-.596	,990	1,018	,028	
		$y/D = .20$		$Q/D^{5/2} = 11.03$			
1	2,750	2.707	-.043	1,077	1,241	,164	
2	4,870	4.548	-.322	,900	,874		-,026
3	6,190	6.389	.199	1,032			
4	7,690	8.230	.540	,891	,886		-,005
5	9,620	9.529	-.091	1,026	1,059	,033	
6	10,750	10.828	.078	,930	,899		-,031
7	12,000	12.127	.127	,984			
8	13,310	13.427	.117	,915	,910		-,005
9	15,000	14.726	-.274	1,014	1,029	,015	
		$y/D = .20$		$Q/D^{5/2} = 12.40$			
1	3,500	3.212	-.288	1,206	1,296	,090	
2	5,690	5.216	-.474	,909	,893		-,016
3	7,250	7.219	-.031	1,080			
4	9,440	9.223	-.217	,915	,907		-,008
5	11,000	10.637	-.363	1,065	1,099	,034	
6	12,500	12.051	-.449	,945	,919		-,026
7	14,000	13.465	-.535	,996			
8	15,500	14.879	-.621	,921	,931		,010

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

	x/D			d/D			
N	TEST	EQ.	DIFF.	TEST	EQ.	DIFF.	DIFF.
		$y/D = .20$		$Q/D^{5/2} = 14.00$			
1	4,000	3.802	-.198	1,305	1,360	,055	
2	6,620	5.996	-.624	,915	,913		-,002
3	8,500	8.189	-.311	1,209			
4	10,620	10.383	-.237	,954	,926		-,028
5	12,120	11.931	-.189	1,080	1,142	,062	
6	14,190	13.479	-.711	,960	,940		-,020
7	15,250	15.027	-.223	,999			
		$y/D = .20$		$Q/D^{5/2} = 15.52$			
1	4,560	4.363	-.197	1,380	1,421	,041	
2	7,250	6.737	-.513	,930	,930		-,000
3	9,750	9.111	-.639	1,287			
4	11,940	11.485	-.455	,960	,945		-,015
5	13,370	13.161	-.209	1,104	1,183	,079	
6	15,250	14.836	-.414	,969	,957		-,012
		$y/D = .20$		$Q/D^{5/2} = 17.02$			
1	5,250	4.916	-.334	1,470	1,481	,011	
2	8,000	7.468	-.532	,966	,945		-,021
3	10,870	10.020	-.850	1,350			
4	13,000	12.572	-.428	,975	,961		-,014
5	14,750	14.374	-.376	1,110	1,223	,113	
		$y/D = .20$		$Q/D^{5/2} = 18.87$			
1	6,000	5.598	-.402	1,500	1,555	,055	
2	9,250	8.370	-.880	,981	,963		-,018
3	12,250	11.142	-1.108	1,355			
4	14,750	13.914	-.836	,990	,980		-,010
		$y/D = .25$		$Q/D^{5/2} = 3.79$			
1	,440	,046	-.394	1,068	1,250	,182	
2	1,810	1.076	-.734	,582	,597		,015
3	2,500	2.106	-.394	,597			
4	3,250	3.136	-.114	,534	,612		,078
5	4,500	3.823	-.677	,600	,685	,085	
6	5,000	4.509	-.491	,573	,635		,062
7	5,750	5.196	-.554	,597			
8	6,370	5.883	-.487	,573	,658		,085
9	7,500	6.569	-.931	,597	,722	,125	
10	8,120	7.256	-.864	,594	,705		,111
11	8,750	7.943	-.807	,630			
		$y/D = .25$		$Q/D^{5/2} = 4.34$			
1	,620	,248	-.372	1,089	1,250	,161	
2	2,120	1.342	-.778	,597	,630		,033
3	2,440	2.437	-.003	,633			
4	3,250	3.531	.281	,576	,640		,064
5	4,440	4.261	-.179	,657	,716	,059	
6	5,190	4.991	-.199	,633	,659		,026
7	6,120	5.720	-.400	,627			
8	6,870	6.450	-.420	,609	,679		,070
9	7,500	7.180	-.320	,642	,731	,089	
10	8,000	7.909	-.091	,633	,695		,062
11	9,000	8.639	-.361	,684			
12	9,690	9.369	-.321	,657	,725		,068

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

N	x/D TEST	x/D EQ.	x/D DIFF.	d/D TEST	d/D EQ.	d/D DIFF.	d/D DIFF.
		y/D = .25		$Q/D^{5/2}$ = 4.34			
13	10.560	10.098	-.462	.714	.789	.075	
14	11.250	10.828	-.422	.678	.768		.090
15	12.000	11.558	-.442	.708			
		y/D = .25		$Q/D^{5/2}$ = 5.36			
1	.690	.622	-.068	1.119	1.250	.131	
2	2.690	1.836	-.854	.726	.685		-.041
3	2.870	3.050	.180	.759			
4	4.250	4.264	.014	.645	.696		.051
5	5.870	5.074	-.796	.762	.789	.027	
6	6.500	5.883	-.617	.747	.714		-.033
7	7.370	6.692	-.678	.780			
8	8.120	7.502	-.618	.753	.728		-.025
9	9.000	8.311	-.689	.777	.790	.013	
10	9.870	9.121	-.749	.774	.745		-.029
11	10.560	9.930	-.630	.801			
12	11.870	10.739	-1.131	.789	.767		-.022
13	12.370	11.549	-.821	.834	.827	-.007	
14	13.310	12.358	-.952	.792	.786		-.006
15	14.000	13.167	-.833	.807			
16	14.750	13.977	-.773	.795	.809		.014
17	15.750	14.786	-.964	.837	.882	.045	
		y/D = .25		$Q/D^{5/2}$ = 6.82			
1	.880	1.158	.278	1.140	1.250	.110	
2	3.250	2.543	-.707	.735	.755		.020
3	4.000	3.928	-.072	.807			
4	5.620	5.314	-.306	.768	.768		.000
5	6.560	6.237	-.323	.822	.877	.055	
6	7.500	7.161	-.339	.798	.780		-.018
7	8.810	8.084	-.726	.822			
8	9.620	9.007	-.613	.813	.795		-.018
9	10.120	9.931	-.189	.828	.866	.038	
10	11.310	10.854	-.456	.816	.807		-.009
11	11.870	11.778	-.092	.849			
12	12.690	12.701	.011	.828	.818		-.010
13	13.750	13.625	-.125	.873	.895	.022	
14	14.750	14.548	-.202	.828	.835		.007
15	15.500	15.471	-.029	.852			
16	16.250	16.395	.145	.834	.849		.015
17	17.120	17.318	.198	.867	.926	.059	
		y/D = .25		$Q/D^{5/2}$ = 8.29			
1	1.120	1.698	.578	1.056	1.250	.194	
2	3.810	3.255	-.555	.801	.816		.015
3	4.750	4.813	.063	.852			
4	6.620	6.370	-.250	.804	.831		.027
5	7.310	7.408	.098	.858	.959	.101	
6	8.750	8.447	-.303	.828	.843		.015
7	9.370	9.485	.115	.861			
8	11.200	10.523	-.677	.828	.857		.029
9	12.000	11.562	-.438	.852	.945	.093	
10	13.000	12.600	-.400	.849	.869		.020
11	14.000	13.638	-.362	.867			
12	14.750	14.676	-.074	.858	.880		.022
13	15.810	15.715	-.095	.888	.971	.083	
14	16.810	16.753	-.057	.852	.895		.043

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

N	x/D TEST	x/D EQ.	x/D DIFF.	d/D TEST	d/D EQ.	d/D DIFF.	d/D DIFF.
		y/D = .25		$Q/D^{5/2}$ = 8.99			
1	1,500	1.955	.455	1,011	1,250	,239	
2	4,060	3.594	-.466	,804	,843		.039
3	4,810	5.234	.424	,882			
4	6,500	6.873	.373	,804	,855		.051
5	8,000	7.966	-.034	,906	,998	,092	
6	9,560	9.059	-.501	,855	,872		.017
7	10,500	10.152	-.348	,888			
8	11,370	11.245	-.125	,864	,882		.018
9	12,620	12.338	-.282	,906	,979	,073	
10	13,810	13.431	-.379	,867	,897		.030
11	14,750	14.524	-.226	,912			
12	15,500	15.617	.117	,873	,908		.035
13	16,620	16.710	.090	,924	1,005	,081	
		y/D = .25		$Q/D^{5/2}$ = 9.41			
1	1,750	2.109	.359	,944	1,250	,306	
2	4,250	3.798	-.452	,834	,850		.016
3	5,250	5.487	.237	1,041			
4	6,810	7.175	.365	,828	,862		.034
5	8,620	8.301	-.319	,897	1,012	,115	
6	10,250	9.427	-.823	,849	,879		.030
7	11,120	10.553	-.567	,900			
8	12,000	11.678	-.322	,849	,889		.040
9	13,500	12.804	-.696	,888	,992	,104	
10	14,750	13.930	-.820	,864	,905		.041
11	15,500	15.056	-.444	,885			
12	16,500	16.181	-.319	,864	,915		.051
		y/D = .25		$Q/D^{5/2}$ = 9.82			
1	2,250	2.260	.010	,978	1,250	,272	
2	4,500	3.996	-.504	,840	,856		.016
3	6,120	5.733	-.387	1,047			
4	7,120	7.470	.350	,837	,868		.031
5	8,880	8.628	-.252	,939	1,024	,085	
6	10,380	9.786	-.594	,852	,884		.032
7	11,380	10.943	-.437	,885			
8	12,380	12.101	-.279	,861	,894		.033
9	13,870	13.259	-.611	,918	1,000	,082	
10	15,380	14.417	-.963	,879	,910		.031
11	16,310	15.575	-.735	,897			
		y/D = .25		$Q/D^{5/2}$ = 10.15			
1	2,750	2.381	-.369	,987	1,250	,263	
2	4,940	4.156	-.784	,849	,861		.012
3	6,120	5.932	-.188	,933			
4	7,620	7.707	.087	,849	,873		.024
5	9,120	8.891	-.229	,966	1,032	,066	
6	10,810	10.074	-.736	,858	,888		.030
7	12,000	11.258	-.742	,900			
8	13,000	12.441	-.559	,858	,899		.041
9	14,500	13.625	-.875	,936	1,008	,072	
10	16,120	14.809	-1.311	,873	,915		.042
		y/D = .25		$Q/D^{5/2}$ = 10.53			
1	2,810	2.520	-.290	,969	1,250	,281	
2	5,060	4.340	-.720	,870	,867		-.003
3	6,080	6.160	.080	,963			

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS.--CONTINUED

	x/D			d/D			
N	TEST	EQ.	DIFF.	TEST	EQ.	DIFF.	DIFF.
		y/D = .25		$Q/D^{5/2}$ = 10.53			
4	7,940	7.980	.040	,861	,879		,018
5	9,370	9.193	-.177	,927	1,044	,117	
6	10,810	10.407	-.403	,882	,892		,010
7	12,060	11.620	-.440	,927			
8	13,500	12.833	-.667	,888	,905		,017
9	15,000	14.047	-.953	,924	1,018	,094	
10	16,060	15.260	-.800	,864	,918		,054
		y/D = .25		$Q/D^{5/2}$ = 11.03			
1	3,060	2.704	-.356	1,066	1,250	,184	
2	5,500	4.583	-.917	,876	,874		-,002
3	6,620	6.461	-.159	,975			
4	8,370	8.340	-.030	,879	,886		,007
5	10,000	9.592	-.408	,972	1,058	,086	
6	11,750	10.844	-.906	,894	,900		,006
7	12,750	12.097	-.653	,924			
8	13,810	13.349	-.461	,891	,910		,019
9	15,500	14.601	-.899	,963	1,028	,065	
10	17,000	15.854	-1.146	,894	,925		,031
		y/D = .25		$Q/D^{5/2}$ = 12.40			
1	3,500	3.207	-.293	1,113	1,296	,183	
2	5,810	5.246	-.564	,876	,893		,017
3	7,500	7.285	-.215	1,047			
4	9,250	9.324	.074	,897	,906		,009
5	11,060	10.684	-.376	,969	1,099	,130	
6	12,310	12.043	-.267	,900	,918		,018
7	13,940	13.402	-.538	,933			
8	15,250	14.762	-.488	,894	,930		,036
9	17,250	16.121	-1.129	,954	1,062	,108	
		y/D = .25		$Q/D^{5/2}$ = 14.00			
1	4,250	3.795	-.455	1,155	1,360	,205	
2	6,620	6.021	-.599	,909	,913		,004
3	8,120	8.248	.128	1,098			
4	10,500	10.474	-.026	,918	,926		,008
5	12,250	11.959	-.291	,996	1,142	,146	
6	13,750	13.443	-.307	,915	,938		.023
7	16,000	14.927	-1.073	,966			
		y/D = .25		$Q/D^{5/2}$ = 15.52			
1	4,870	4.353	-.517	1,182	1,421	,239	
2	7,620	6.757	-.863	,936	,930		-,006
3	10,060	9.162	-.898	1,173			
4	11,500	11.567	.067	,942	,942		.000
5	13,500	13.170	-.330	,993	1,182	,189	
6	15,250	14.773	-.477	,933	,955		.022
7	17,620	16.376	-1.244	,963			
		y/D = .25		$Q/D^{5/2}$ = 18.10			
1	5,500	5.300	-.200	1,287	1,524	,237	
2	9,000	8.007	-.993	,939	,956		.017
3	12,000	10.714	-1.286	1,233			
4	13,250	13.421	.171	,993	,969		-,024
5	16,000	15.226	-.774	1,017	1.250	,233	

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

N	x/D TEST	x/D EQ.	x/D DIFF.	d/D TEST	d/D EQ.	d/D DIFF.	d/D DIFF.
		y/D = .25		$Q/D^{5/2}$ = 19.54			
1	6,250	5,829	-.421	1,293	1,582	,289	
2	10,000	8,705	-1.295	1,014	,969		-,045
3	12,750	11,580	-1.170	1,197			
4	14,500	14,456	-.044	1,041	,983		-,058
5	17,500	16,373	-1.127	1,038	1,286	,248	
		y/D = .30		$Q/D^{5/2}$ = 4.34			
1	,370	,259	-.111	1,191	1,300	,109	
2	2,060	1,394	-.666	,618	,610		-,008
3	2,690	2,529	-.161	,651			
4	3,690	3,664	-.026	,594	,622		,028
5	4,750	4,379	-.371	,669	,696	,027	
6	5,250	5,094	-.156	,612	,635		,023
7	6,000	5,809	-.191	,642			
8	6,750	6,525	-.225	,624	,649		,025
9	7,440	7,240	-.200	,690	,700	,010	
10	8,370	7,955	-.415	,636	,667		,031
11	8,940	8,670	-.270	,669			
12	9,870	9,385	-.485	,654	,686		,032
13	10,750	10,100	-.650	,702	,742	,040	
14	11,560	10,815	-.745	,681	,712		,031
15	12,000	11,530	-.470	,696			
16	12,690	12,245	-.445	,681	,736		,055
17	13,500	12,960	-.540	,720	,802	,082	
		y/D = .30		$Q/D^{5/2}$ = 5.36			
1	,500	,632	.132	1,206	1,300	,094	
2	2,370	1,885	-.485	,684	,670		-,014
3	3,000	3,138	.138	,714			
4	4,060	4,391	.331	,660	,680		,020
5	5,250	5,180	-.070	,738	,769	,031	
6	6,060	5,970	-.090	,690	,694		,004
7	6,440	6,759	.319	,708			
8	7,310	7,548	.238	,690	,704		,014
9	8,000	8,338	.338	,762	,763	,001	
10	8,940	9,127	.187	,705	,717		,012
11	9,560	9,916	.356	,738			
12	10,690	10,706	.016	,720	,732		,012
13	11,500	11,495	-.005	,765	,794	,029	
14	12,370	12,284	-.086	,735	,749		,014
15	12,750	13,074	.324	,771			
16	13,750	13,863	.113	,744	,765		,021
17	14,500	14,652	.152	,792	,828	,036	
		y/D = .30		$Q/D^{5/2}$ = 6.82			
1	,690	1,166	.476	1,239	1,300	,061	
2	2,750	2,588	-.162	,756	,745		-,011
3	3,620	4,010	.390	,804			
4	4,870	5,431	.561	,750	,757		,007
5	6,500	6,327	-.173	,840	,869	,029	
6	7,310	7,223	-.087	,783	,772		-,011
7	7,750	8,118	.368	,810			
8	8,750	9,014	.264	,786	,781		-,005
9	9,750	9,910	.160	,843	,856	,013	
10	10,620	10,805	.185	,804	,793		-,011
11	11,500	11,701	.201	,852			
12	12,250	12,597	.347	,810	,805		-,005

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS--CONTINUED

N	x/D TEST	x/D EQ.	x/D DIFF.	d/D TEST	d/D EQ.	d/D DIFF.	d/D DIFF.
		y/D = .30		Q/D$^{5/2}$ = 6.82			
13	13.500	13.492	-.008	.870	.883	.013	
14	14.440	14.388	-.052	.816	.822		.006
15	15.000	15.284	.284	.873			
		y/D = .30		Q/D$^{5/2}$ = 8.29			
1	1.000	1.703	.703	1.188	1.300	.112	
2	3.370	3.295	-.075	.807	.813		.006
3	4.250	4.887	.637	.855			
4	5.750	6.479	.729	.795	.825		.030
5	7.120	7.482	.362	.900	.957	.057	
6	8.120	8.484	.364	.822	.838		.016
7	9.120	9.487	.367	.846			
8	9.750	10.490	.740	.825	.847		.022
9	10.500	11.492	.992	.882	.935	.053	
10	11.690	12.495	.805	.825	.859		.034
11	12.870	13.498	.628	.864			
12	13.750	14.501	.751	.849	.873		.024
13	14.750	15.503	.753	.906	.962	.056	
		y/D = .30		Q/D$^{5/2}$ = 8.99			
1	1.190	1.959	.769	1.155	1.300	.145	
2	3.810	3.632	-.178	.825	.843		.018
3	5.000	5.305	.305	.885			
4	6.060	6.978	.918	.825	.854		.029
5	7.750	8.031	.281	.954	.998	.044	
6	8.810	9.085	.275	.855	.869		.014
7	9.870	10.139	.269	.864			
8	10.690	11.192	.502	.855	.879		.024
9	12.000	12.246	.246	.912	.977	.065	
10	12.810	13.300	.490	.855	.892		.037
11	14.060	14.353	.293	.885			
12	15.000	15.407	.407	.900	.906		.006
		y/D = .30		Q/D$^{5/2}$ = 9.41			
1	1.310	2.113	.803	1.128	1.300	.172	
2	4.250	3.834	-.416	.834	.850		.016
3	5.250	5.555	.305	.912			
4	6.560	7.277	.717	.834	.861		.027
5	8.060	8.361	.301	.987	1.009	.022	
6	9.440	9.445	.005	.861	.875		.014
7	10.250	10.530	.280	.909			
8	11.440	11.614	.174	.861	.886		.025
9	13.120	12.698	-.422	.918	.989	.071	
10	14.370	13.783	-.587	.858	.903		.045
11	15.250	14.867	-.383	.894			
		y/D = .30		Q/D$^{5/2}$ = 9.82			
1	1.440	2.263	.823	1.113	1.300	.187	
2	4.500	4.031	-.469	.849	.856		.007
3	5.370	5.800	.430	.927			
4	6.940	7.569	.629	.849	.867		.018
5	8.000	8.683	.683	.990	1.020	.030	
6	9.870	9.797	-.073	.885	.881		-.004
7	10.750	10.911	.161	.912			
8	12.000	12.026	.026	.882	.892		.010
9	13.310	13.140	-.170	.924	.997	.073	
10	14.500	14.254	-.246	.897	.906		.009

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS.--CONTINUED

	x/D			d/D			
N	TEST	EQ.	DIFF.	TEST	EQ.	DIFF.	DIFF.
		$y/D = .30$		$Q/D^{5/2} = 10.15$			
1	1.690	2.383	.693	1.095	1.300	.205	
2	4.620	4.190	-.430	.852	.861		.009
3	5.690	5.997	.307	.939			
4	7.060	7.804	.744	.861	.872		.011
5	8.690	8.942	.252	1.005	1.032	.027	
6	10.190	10.080	-.110	.888	.887		-.001
7	11.000	11.219	.219	.930			
8	12.370	12.357	-.013	.900	.897		-.003
9	13.750	13.495	-.255	.930	1.006	.076	
		$y/D = .30$		$Q/D^{5/2} = 10.53$			
1	1.940	2.522	.582	1.074	1.300	.226	
2	4.690	4.373	-.317	.900	.867		-.033
3	5.870	6.224	.354	.945			
4	7.310	8.075	.765	.870	.878		.008
5	8.810	9.241	.431	1.020	1.043	.023	
6	10.370	10.407	.037	.888	.892		.004
7	11.500	11.572	.072	.924			
8	12.620	12.738	.118	.888	.902		.014
9	14.440	13.904	-.536	.945	1.017	.072	
		$y/D = .30$		$Q/D^{5/2} = 11.03$			
1	2.370	2.705	.335	1.065	1.300	.235	
2	4.870	4.614	-.256	.885	.874		-.011
3	6.250	6.522	.272	.954			
4	7.690	8.431	.741	.873	.886		.013
5	9.000	9.633	.633	1.026	1.057	.031	
6	10.500	10.836	.336	.894	.898		.004
7	11.810	12.038	.228	.945			
8	12.940	13.240	.300	.894	.909		.015
9	14.750	14.442	-.308	.960	1.027	.067	
		$y/D = .30$		$Q/D^{5/2} = 12.40$			
1	3.250	3.206	-.044	1.122	1.300	.178	
2	5.370	5.273	-.097	.888	.893		.005
3	7.000	7.340	.340	1.020			
4	8.620	9.407	.787	.873	.905		.032
5	10.120	10.709	.589	1.083	1.097	.014	
6	11.810	12.011	.201	.912	.917		.005
7	13.250	13.313	.063	.975			
8	14.560	14.615	.055	.912	.929		.017
		$y/D = .30$		$Q/D^{5/2} = 14.00$			
1	4.250	3.791	-.459	1.218	1.360	.142	
2	6.370	6.043	-.327	.909	.913		.004
3	8.250	8.295	.045	1.080			
4	10.000	10.547	.547	.927	.925		-.002
5	11.750	11.966	.216	1.125	1.141	.016	
6	13.870	13.384	-.486	.924	.939		.015
7	15.250	14.803	-.447	.990			
		$y/D = .30$		$Q/D^{5/2} = 15.52$			
1	5.000	4.347	-.653	1.335	1.421	.086	
2	7.250	6.775	-.475	.936	.930		-.006
3	9.120	9.202	.082	1.245			

TABLE 2.--SUMMARY OF DATA AND AGREEMENT WITH EQUATIONS.--CONTINUED

	x/D			d/D			
N	TEST	EQ.	DIFF.	TEST	EQ.	DIFF.	DIFF.
		$y/D = .30$		$Q/D^{5/2} = 15.52$			
4	11,870	11.630	-.240	,960	,945		-.015
5	13,870	13.159	-.711	1,131	1,185	,054	
		$y/D = .30$		$Q/D^{5/2} = 16.67$			
1	5,500	4,767	-.733	1,401	1,467	,066	
2	8,500	7,328	-1.172	,954	,942		-.012
3	10,870	9,889	-.981	1,260			
4	13,250	12.450	-.800	,975	,957		-.018
5	14,750	14.063	-.687	1.140	1,212	,072	